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ORIGINAL INNOVATION

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Reliability polynomial chaos metamodel for the dynamic behaviour of reinforced concrete bridges

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Abstract

The approximation of complex engineering problems and mathematical regressions serves as the authentic inspiration behind the artificial intelligence metamodeling methods. Among these methods, polynomial chaos expansion, along with artificial neural networks, has emerged at the forefront and become the most practical technique. Previous studies have highlighted their robust capabilities in solving complex problems and their wide utilization across numerous applications, particularly in structural analysis, optimization design problems, and predictive models of uncertainty outcomes. The aim of this article is to present a methodology that introduces their implementation of for structural engineering, primarily focusing on reinforced concrete bridges. The proposed approach consists of demonstrating the applicability of the polynomial chaos to evaluate the dynamic behavior of two-span reinforced concrete bridges through a predictive model of natural vibration properties for eigenvalues modal analysis. Subsequently, response spectral method is conducted according to the Moroccan guide for bridge seismic design and the prescription of the EUROCODE 8 within the context of reliability assessment using Monte Carlo simulation. The efficacy of the proposed approach is illustrated by a comparison between the predicted vibration properties and the resulting values obtained through finite element modal analysis and artificial neural networks. The polynomial chaos process is based on a collected dataset of multiple reinforced concrete bridges sourced from technical studies offices and the Regional Administration of the East, affiliated with the Moroccan Ministry of Equipment and Water. Finally, this work contributes to the field by enhancing predictive modeling and reliability evaluation for bridge engineering using artificial intelligence metamodels.

Keywords: Reinforced concrete bridges, Polynomial chaos expansion, Structural reliability, Bridge engineering, Structural dynamics, Monte Carlo simulation

1 Introduction

1.1 Overview of the literature

Polynomial chaos expansion (PCE) has been widely applied in various fields, including finance, statistics, meteorology, and more engineering domains. The method is found to be a valuable tool for dealing with probabilistic outcomes and allows the making of prediction models, uncertainty quantification, and sensitivity analysis. Polynomial chaos expansion is used to model and analyze systems influenced by uncertainty. Originally introduced by Norbert Wiener (1938) in the context of stochastic processes, the core idea of the polynomial chaos method is to represent a random process as a series of orthogonal polynomials of random variables, providing a systematic framework for dealing with uncertainties. The versatility and robustness of the metamodel technique have led to its widespread application in civil engineering. It has been employed to model strength properties, predict structural responses under uncertain loading conditions, assess the reliability of structures, and perform dynamic analysis of systems subject to random vibrations and ground motion excitations. Relevant studies such as those by Sochala et al. (2019), have demonstrated its utility where the expansion framework is used for geosciences to estimate the uncertainties of hurricane-induced storm surges by illustrating a predictive model based on a simulation of the flooding caused by Hurricane Gustav in 2008. The research by Saassouh et al. (2011) employs the polynomial method to model uncertainties in physical parameters governing the corrosion induction in steel reinforcement of concrete structures. The relevance of the study has been demonstrated on a highway bridge structure and compared to the probabilistic modeling findings using Monte Carlo simulation. For bridge engineering, polynomial chaos expansion have recently been implemented in multiple applications. Notably, the research of Pinghe Ni et al. (2019), which represents a system of output characterized by the natural vibration properties of dynamic responses of bridge structures, the obtained results present high efficiency and good accuracy when compared to Monte Carlo simulation and First-Order Second-Moment methods. In the same line, linked to safety concept and probabilistic modeling, the study of Mosleh et al. (2018) focuses on the stochastic response of concrete bridges using a generalized polynomial chaos expansion while accounting for uncertainty in the stiffness of bearings and abutments. Pinghe Ni et al. (2023) propose a reliability assessment strategy by combining the polynomial chaos method and simulation techniques for bridges under diverse loading conditions. The expansion is applied to approximate the performance functions and to compute the probability of failure. The methodology was applied to three structural application cases: a truss bridge, a beam under a moving load, and a reinforced concrete bridge subjected to a ground motion excitation. The obtained results yield an accurate estimation using Monte Carlo simulation. The work of Novak et al. (2022) presents a semi-probabilistic methodology for the assessment of structures that is based on Gram–Charlier and polynomial chaos expansion. The proposed approach is applied for the determination of the load-bearing capacity of an existing prestressed concrete bridge. Additionally, The study of Yue Shang et al. (2024) proposes an adapted sensitivity method combined with polynomial chaos, which is used to approximate the output and alleviate the calculation cost in sensitivity analysis of a truss structure and a dynamic train-track-bridge system. Related to structural reliability and uncertainty quantification, the surrogate technique has been adopted in a certain number of studies. Jun Xu et al. (2019) proposes a combined method of polynomial chaos expansion with Voronoi cells and a dimension reduction technique for structural reliability analysis. The model was validated with four numerical examples where the findings show the efficiency of meta-models to perform structural reliability analysis with low computational cost. In the same context, Shi et al. 2011 propose a mechanism for reliability analysis that approximates the limit state function of the structural problem using the polynomial chaos metamodel. Then, the outcomes of the reliability analysis were consistent with the results of the Monte Carlo simulation, proving high efficiency. The study of Ming Chen et al. (2022) consists of employing polynomial chaos expansion for uncertainty modeling and sensitivity analysis. The authors present two numerical tests related to Fortini's clutch with the Ishigami function and two cases of study considering a bar truss system and a roof truss. The obtained results are compared to Monte Carlo simulation, which presents good accuracy to solve nonlinear complex problems in structural engineering. As seen from the available literature, polynomial chaos has emerged as a powerful predictive tool for solving complex structural problems, in an alternative way to artificial neural networks, which are particularly advantageous due to their ability to model nonlinear relationships and handle large datasets effectively. As an example, the study of Hicham Lamouri et al. (2024) approves the effectiveness of neural networks in combination with the First-Order Reliability Method (FORM) and Monte Carlo Simulation to predict the flexural stress and estimate the failure probability for prestressed bridge beams. Then, the polynomial chaos expansion provides too a systematic path to handle uncertainties, perform sensitivity analysis, and approximate complex responses, which offers distinct advantages in terms of computational efficiency and accuracy in uncertainty quantification. Therefore, in this study, affiliated with the Civil Engineering and Construction Laboratory of CEDOC-EMI, important aspects of reliability and structural dynamics for bridge engineering are discussed within the concept of metamodels and artificial intelligence techniques. The paper builds upon presenting the prediction aspect of polynomial chaos expansion aligned with reliability dependent on dynamic analysis for bridges according to national and international earthquake design standards. While providing a better understanding by comparing the obtained findings to neural networks approximation and structural finite element model. The novelty of this work lies in the innovative combination of polynomial chaos and dynamic spectral analysis within the context of reliability engineering, leveraging the strengths of the metamodel to create a more robust and accurate predictive tool. This integrated approach not only enhances the efficiency and accuracy of reliability assessment but also provides a comprehensive framework for handling complex uncertainties in structural responses, especially for bridges. The effectiveness of the proposed methodology will be demonstrated through its application to a two-span reinforced concrete bridge structure, for a modal eigen value estimation, highlighting its potential to significantly improve the dynamic evaluation processes in bridge engineering.

1.2 Organization of the paper

The rest of this paper is organized as follows. In the Methods section, the theoretical background of the polynomial chaos expansion is presented and the study methodology for the practical structural application is introduced. In the Results section, the structural configuration of the reinforced concrete bridge case study and the data set used are presented, along with a detailed analysis of the eigen values obtained from the bridge dynamic model. The finite element modeling and analysis are discussed, followed by a reliability analysis based on the response surface methodology. The Discussion section provides an interpretation of the results, and the Conclusion summarizes the key findings and implications of the study.

2 Methods

2.1 Polynomial Chaos Expansion: Brief theoretical presentation

Polynomial chaos expansion (PCE), also known as Wiener Expansion, is admitted as a metamodel or surrogate-based method to determine the evolution of a system when a probabilistic uncertainty in its parameters exists. The general concept and the underlying theory of the expansion method were introduced by Nobbert Wiener in (1938) and more elaborated by the Cameron and Martin theorem in (1947). It is considered a numerical algorithm that represents the parameters of interest as a polynomial development in terms of certain basis functions. These basis functions are typically orthogonal polynomials, such as Hermite, Legendre, or Jacobi polynomials. The key advantage behind the introduction of these polynomials is their desirable mathematical properties that facilitate efficient representation and computation. The major property is orthogonality, which simplifies the computations of the system response in terms of the expansion coefficients while respecting the distribution of the parameters. For more detail, see (Xiu 2002), and some numerical examples are provided in (Bruno Sudret 2007).

The general process of polynomial chaos expansion for uncertainty quantification is sketched in Fig. 1. Quantifying the response of a model involves defining the probability density distribution, statistically analyzing the results, predicting the desired quantities and estimating the probability of failure. Therefore, the objective of polynomial chaos extension is to simulate the response of a stochastic output variable as a function of stochastic input variables. The general mathematical form of a polynomial chaos expansion for a one-dimensional model response f(X) can be written as shown in (1):

$$Y = f(\mathrm{X}) = \sum_{\mathrm{i}=0}^{\mathrm{N}} \alpha_{\mathrm{i}} \varphi_{\mathrm{i}}(\mathrm{X}) \tag{1}$$

where Y represents the random variable to be modelled describing the output. X are the input variables. N represents the degree orders of the expansion while $\varphi_{\mathrm{i}}(\mathrm{X})$ are orthogonal polynomial basis functions while respecting the probability distribution of the random parameters to ensure accurate representation and efficient computation. They constitute a basis of the probabilistic space associated with the input random variables (Kersaudy 2013) α_{i} are the extension coefficients. These are determined using the orthogonality properties of chaos polynomials and can be computed by projection methods.

When a random function is developed as a series of orthogonal polynomials, this simplifies the calculations and makes the analysis more tractable. The families of

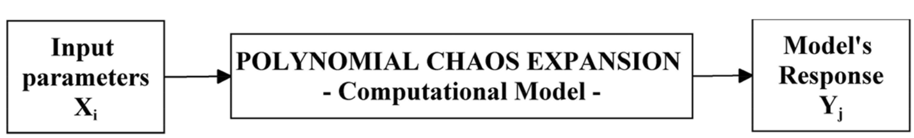

Fig. 1 The general process of Polynomial Chaos Expansion (Bruno Sudret 2007)

orthogonal polynomials depend mainly on the type of distribution of the random variable and must satisfy orthogonality relations with respect to a certain probability measure according to the following condition (2):

$$\int P_i(X)P_j(X)w(X) = 0 \, for \; i \neq j \tag{2}$$

where w(X) is a weight function. This property is essential because it allows a complex function to be decomposed into a series of mutually independent terms, making it easier to calculate the coefficients of the polynomial extension. The commonly used orthogonal polynomial families are given as follows (3–7):

- Monomial defined over the interval of the variables domain [a, b].

$$P_n(X) = X^n \tag{3}$$

- Legendre Polynomial defined over the interval [-1, 1].

$$P_n(X) = \frac{1}{2^n n!}\frac{d^n}{dX^n}\left[(X-1)^n\right] \tag{4}$$

- Hermit Polynomial defined over the interval [-∞, ∞].

$$P_n(X) = (-1)^n e^{X^2}\frac{d^n}{dX^n}\left[e^{X^2}\right] \tag{5}$$

- Laguerre Polynomial defined over the interval [0, ∞].

$$P_n(X) = \frac{e^X}{n!}\frac{d^n}{dX^n}\left[e^{-X}X^n\right] \tag{6}$$

- Jacobi Polynomial defined over the interval [-1, 1] with Beta function coefficients (α, β).

$$P_n(X) = \frac{(-1)^n}{2^n n!}\frac{d^n}{dX^n}\left[(1-X)^\alpha(1+X)^\beta\left(1-X^2\right)^n\right] \tag{7}$$

A detailed background on the theoretical foundations of polynomial chaos expansion is provided by O'Hagan (2013). For a generalization over multidimensional vectors with multiple inputs, the expansion of the polynomial chaos can be represented as follows (8).

$$Y = f(X) = \sum\nolimits_{i\epsilon I}\alpha_i(\prod\nolimits_{k=1}^{d}\varphi(X_k)) = \sum\nolimits_{i_1}^{N_1}\sum\nolimits_{i_2}^{N_2}..\sum\nolimits_{i_d}^{N_d}\alpha_{i_{1,i_2,\ldots,i_d}}\varphi_{i_1}(X_1)\varphi_{1_2}(X_2)..\varphi_{i_d}(X_d) \tag{8}$$

where I is the set of all possible multi-indices $i = (i_1, i_1 \ldots, i_d)$ satisfying $0 \leq i_k \leq N_k$ for $k = (1, 2, \cdots, d)$. α_i are the expansion coefficients corresponding to the multi-index $i = (i_1, i_1 \cdots, i_d)$. $\varphi_{i_k}(X_k)$ are the orthogonal polynomials chosen as basis functions for each input parameter X_k. N_k are the orders of the expansions for each input parameter, indicating the highest degree of the polynomial terms included in the expansions.

2.2 Study methodology

The principal approach of the study is to perform a comprehensive reliability assessment for seismic stress failure using the response spectral method on a reinforced concrete bridge. The first step involves analyzing the dynamic behavior of the bridge structure, which is modeled as a two-degree-of-freedom system in the longitudinal vibration direction. This dynamic behavior is characterized by the natural periods of vibration of the structure. To approximate these natural periods, the study employs polynomial chaos expansion for an approximated eigenvalues analysis. This method leverages the predefined characteristics and the data set collected from various bridge configurations to ensure accuracy and relevance in the analysis. Following this, the study transitions to performing a reliability analysis based on the dynamic spectral method. This phase is crucial defining the limit state function of the reliability problem, as it involves defining regional seismic data and constructing a response spectrum curve. This last one, which is fundamental to seismic analysis, captures the peak response of the structure under different frequencies of ground motion. Practically, it is defined by the national and international standards of seismic design. Thus, by integrating regional seismic data, the study ensures that the analysis is adapted to the specific seismic conditions of the bridge site area in question. This step is vital for developing a consistent and realistic assessment of the bridge's reliability under seismic stress. The overall approach aims to provide a robust framework for understanding and ameliorating seismic analysis of reinforced concrete bridges while implementing artificial intelligence techniques such as polynomial chaos expansion for the present study while considering the probabilistic aspect of design variables. The flowchart given in Fig. 2 presents the main steps of the overall thinking of the paper methodology.

3 Results

3.1 Structural configuration

The study consists to describe and to apply metamodeling based on the polynomial chaos surrogate method to evaluate the dynamic behavior of a set of bridges located in the province of Oujda Angad, East of Morocco, which is known for its modern seismicity according to the Moroccan seismic regulations code (RPS 2011). A bridge is always vibrating under the effects of several factors, including earthquakes. As defined previously in Sect. 2.2, the objective is to provide a predictive metamodel for computing the natural periods of longitudinal vibration of reinforced concrete bridges with two spans precisely. The determination of the structural natural period can be influenced by several multiple parameters, notably the bridge deck and the support parameters, as considered by some international seismic designs such as the European standard (EUROCODE8 1998) and the Algerian seismic regulations code (RPA 2003). The choice of the number of input variables depends on the complexity of the problem and the speed of compilation. As the number of structural elements making up the bridge is very large (girders, slabs, sidewalks, cornices, etc.) and their dimensions are very vast, it is useful to summarize all this into two parameters: the deck with pier masses, considering all the equipment weights M, and the stiffness K of the pier and bearings support system. To do so, these two parameters are set as

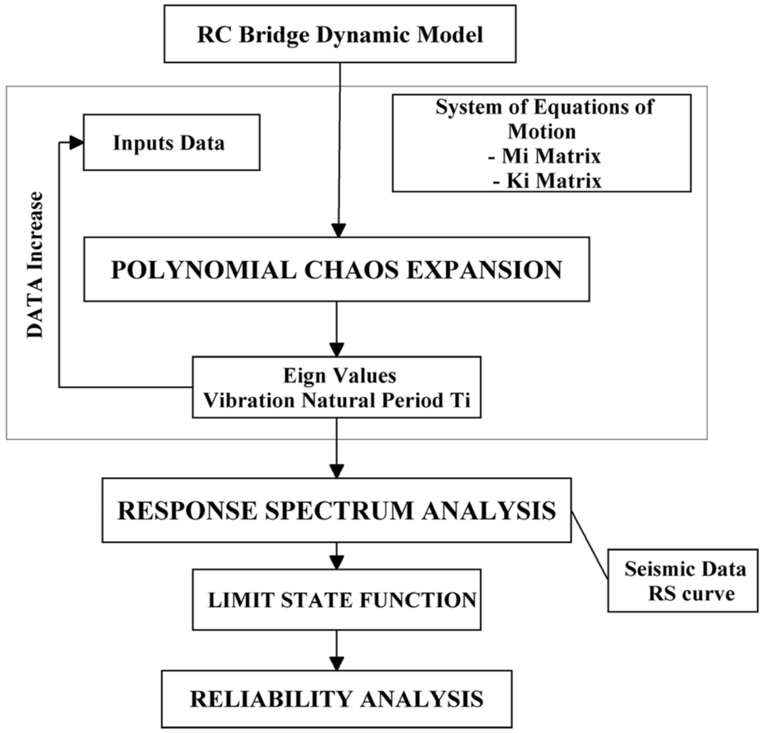

Fig. 2 Methodology flowchart

Fig. 3 Perspective view of the Reinforced Concrete Bridge Case Study—1

inputs in order to arrive at the output calculation, which is the natural period T. The structural configuration of the case study is a typical two-span reinforced concrete bridge composed of four beams with a span length of 18 m and a width of 11.22 m. The pier system is composed of a pier cap and three columns with a diameter of 1 m for each. Figs. 3 and 4 display a perspective view of the structure. The transversal profile of the bridge is given in Fig. 5, while Fig. 6 shows the longitudinal profile.

Fig. 4 Perspective view of the Reinforced Concrete Bridge Case Study—2

3.2 Eigen values analysis

The bridge system rests on a single pier with three columns in the middle and two support bearings at the ends. The bridge is modeled as a two-degree-of-freedom system. The deck moves as a rigid body with mass M_2 and is connected to the pier by the bearing as a spring-dumper system (K_a,C_a). Similarly, at both ends, the girders are supported by bearings. The pier with mass M_1 has a stiffness K_p and damping coefficient C_p. Therefore, for the present study, the pier and deck are assumed to move in a plane in a longitudinal direction only. Fig. 7 illustrates the adopted dynamic model of the bridge. The modal eigenvalues analysis consists of solving the system at a free and undamping vibration. The formulation of the system of motion using the Lagrange principle (Landau 1994) is given as follows in (9), where T and V are the kinetic and potential energies, respectively. The derivative development is expressed in (10) and (11).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial \dot{x}} = 0; L = T - V \tag{9}$$

$$\begin{cases} T = \frac{1}{2}M_1\dot{x}_1^2 + \frac{1}{2}M_2\dot{x}_2^2 \\ V = \frac{1}{2}K_p{x_1}^2 - \frac{1}{2}K_a(x_1 - x_2)^2 - \frac{3}{2}K_a{x_2}^2 \end{cases} \tag{10}$$

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial \dot{x}_1} = M_1\dot{x}_1 + \left(K_p + K_a\right)x_1 - K_a x_2 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial \dot{x}_2} = M_2\dot{x}_2 - K_a x_1 + 3K_a x_2 \end{cases} \tag{11}$$

By setting the equations above in matrix form, the resulting system of equations is given as follows in (12), where M is the matrix of mass and K is the stiffness.

$$\mathrm{M}\ddot{X}_i + KX_i = 0; M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}; K = \begin{pmatrix} K_p + K_a & -K_a \\ -K_a & 3K_a \end{pmatrix} \tag{12}$$

Then, the calculation of natural periods consists of resolving the determinant of the following system (13).

$$\left|K - M\omega^2\right| = 0 \tag{13}$$

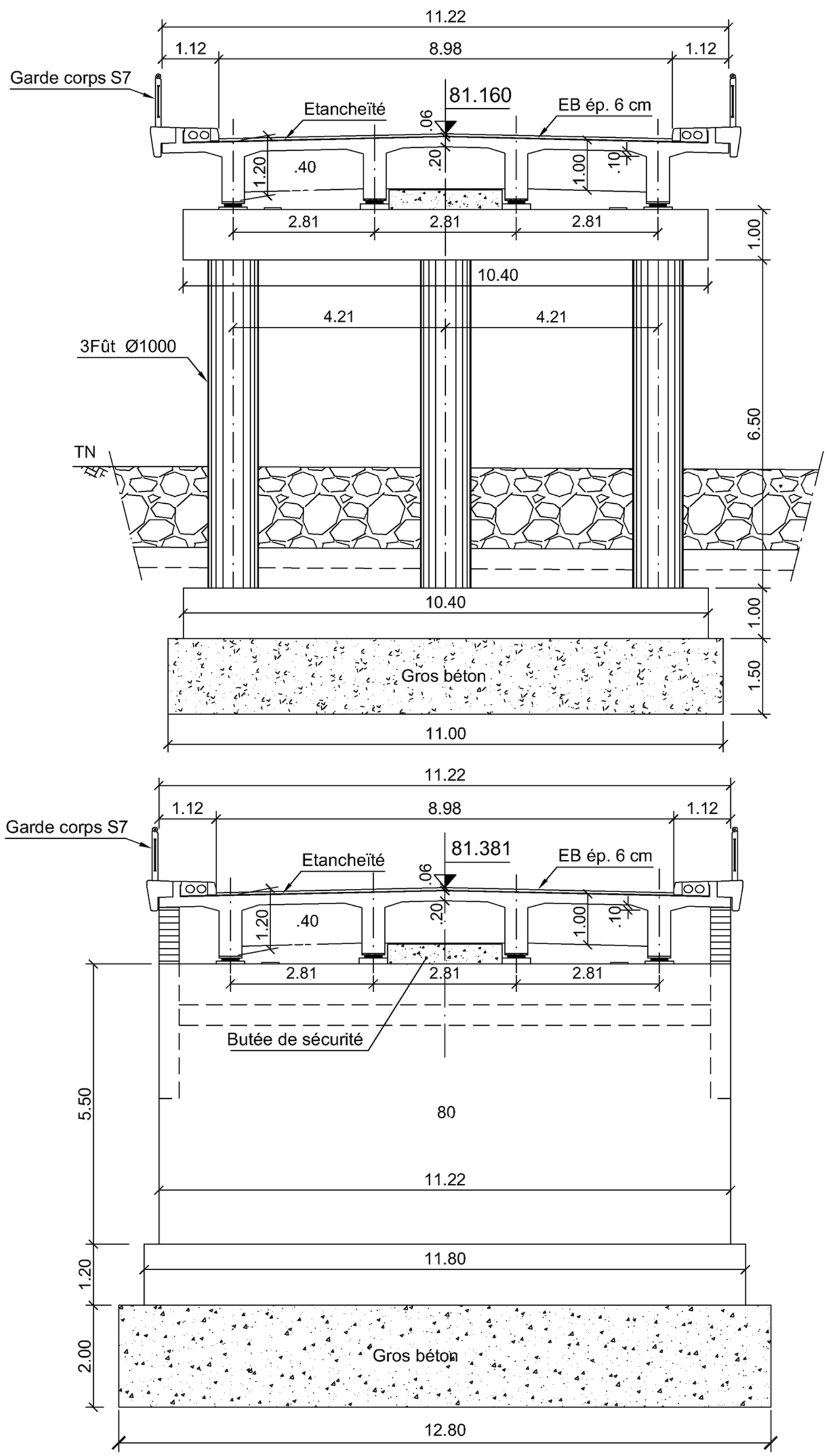

Fig. 5 Transversal profile of the Reinforced Concrete Bridge Case Study (Pier and abutment)

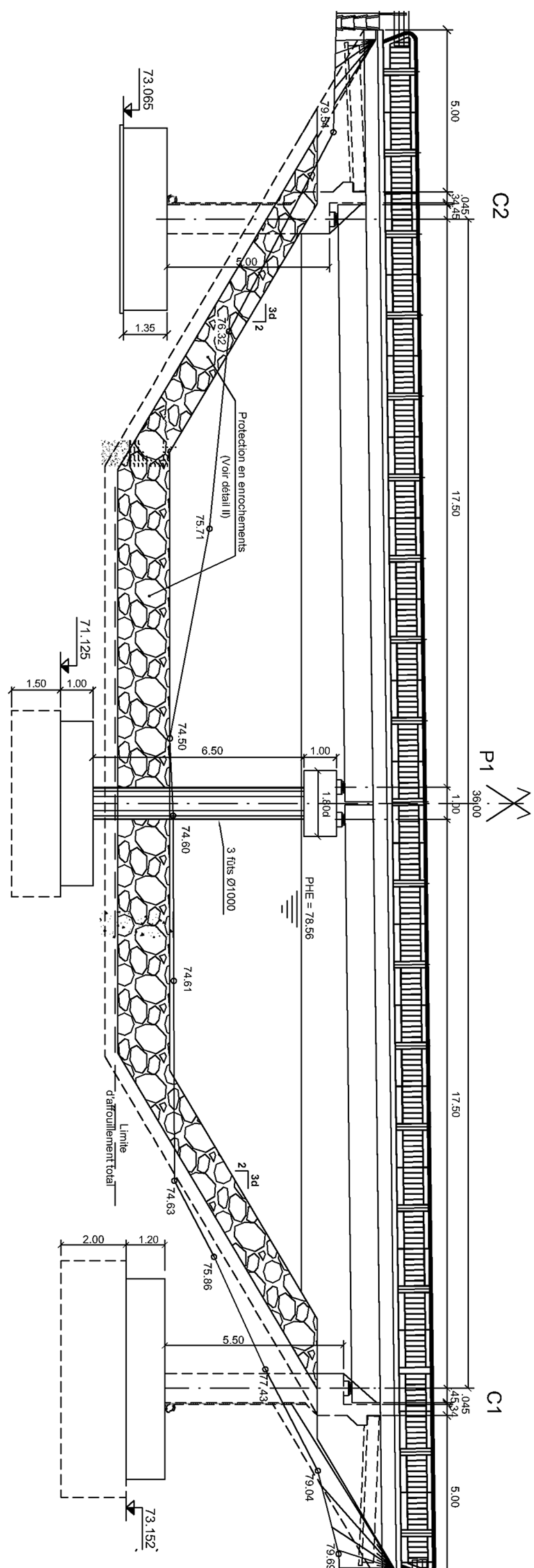

Fig. 6 Longitudinal profile of the Reinforced Concrete Bridge Case Study

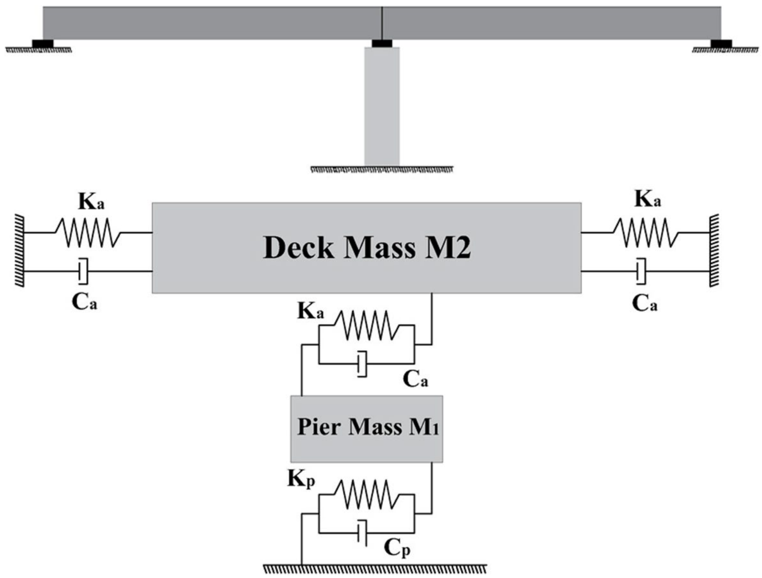

Fig. 7 Dynamic model of the bridge

Table 1 Data parameters and variation's ranges

Type	Description	Unit	Variation interval
INPUTS	Number of columns	-	$n_C = [1; 2; 3]$
	Cross section Inertia of one column	m^4	$0.003 \leq I_c \leq 0.785$
	Height of the column	m	$4 \leq H_c \leq 10$
	Mass of the deck	T	$1454 \leq M_2 \leq 2497$
	Mass of the pier	T	$1.785 \leq M_1 \leq 20.135$
	Bearing stiffness	kN/m	$18404 \leq K_a \leq 20000$
	Pier stiffness	MN/m	$1.28 \leq K_p \leq 1287.9$
OUTPUT	Period of 1th mode	s	$1.033 \leq T_1 \leq 1.522$
	Period of 2th mode	s	$0.014 \leq T_2 \leq 0.137$

3.3 Data set and results

The implementation of the polynomial chaos method first requires a consistent database. The model was created utilizing previous experimental design data from various reinforced concrete bridges with the same type of case study. These data are collected from technical offices and the Regional Administration of East, affiliated to the Moroccan Ministry of Equipment and Water. The information about the required parameters and their ranges of variation is summarized in Table 1. The expansion process was conducted using MATLAB. The definition of the appropriate degree of the expansion was carried out through numerical simulations using the following case orders: one, two, and three. The analysis of the data regression of each order from the expansion phases, led

Table 2 MSE and Regression comparison of PCE orders

Order 1		Order 2		Order 3	
MSE	R^2	MSE	R^2	MSE	R^2
0.0151	0.155	1.8e-04	0.98	6e-05	0.99

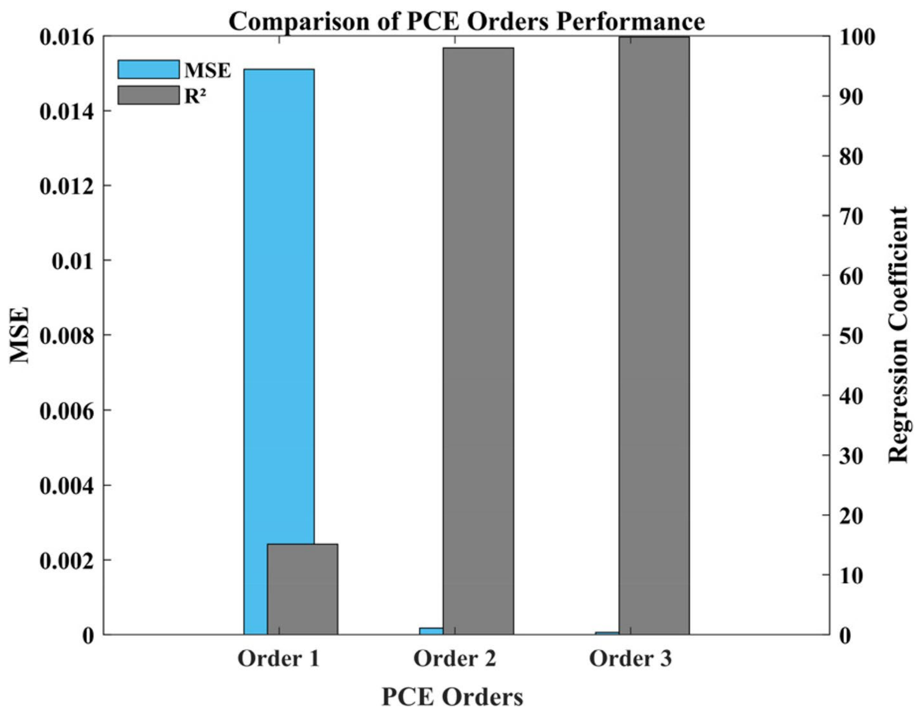

Fig. 8 Performance of PCE orders

to the adoption of a polynomial approximation of order 3 regarding the alignment of the resulting regression fitness with the ideal prediction line, characterized by a regression coefficient (R) according to the following formula (14).

$$R = \sqrt{1 - \frac{\sum_{k}^{N}\left(\widehat{Y_k} - Y_k\right)^2}{\sum_{k}^{N}(Y - Y_k)^2}} \tag{14}$$

In further, the obtained values of the regression coefficients and the mean square errors are shown in Table 2, and are illustrated in Fig. 8. It is worth noting that the mean square error (MSE) is calculated to obtain a mean value by squaring the difference between the targets Y_k and the predicted values $\widehat{Y_k}$, divided by the data's number, as expressed in Eq. (15).

$$MSE = \frac{1}{N}\sum_{k}^{N}\left(\widehat{Y_k} - Y_k\right)^2 \tag{15}$$

The comparison between the three polynomial orders for the corresponding outputs is given in Figs. 9 and 10. In addition, Figs. 11 and 12 show the data's regression for the

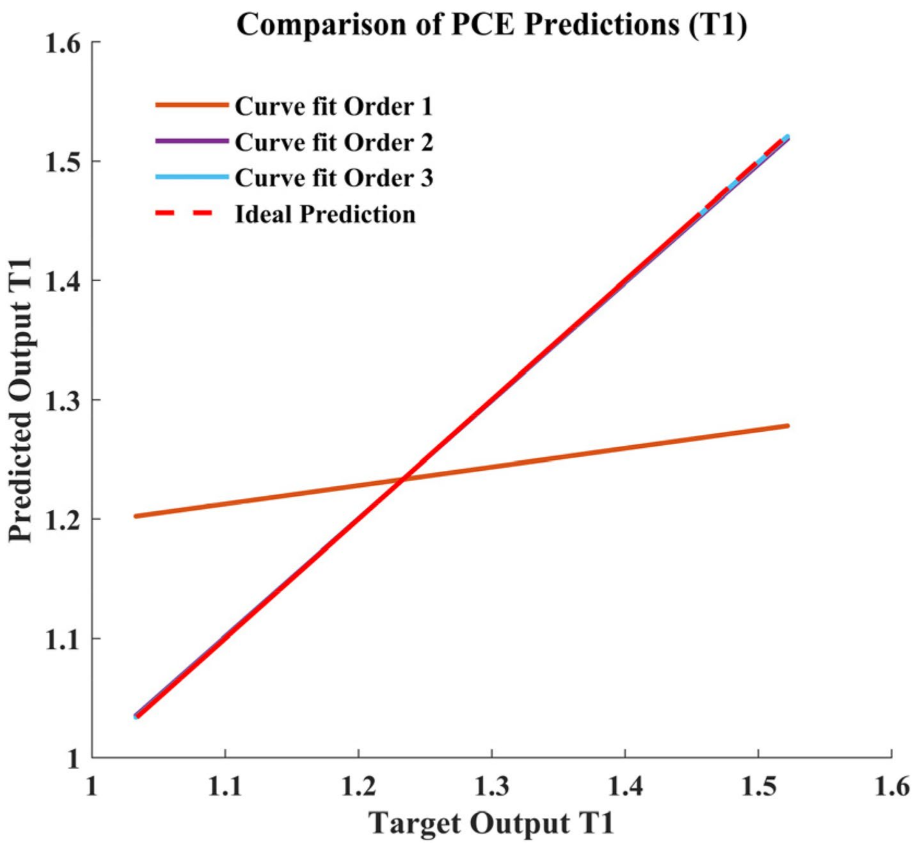

Fig. 9 PCE Orders comparison for T1

adopted polynomial order of the corresponding eigenvalues. As follows, their predicted values for the case study inputs are detailed in Table 3. The computation process of the polynomial chaos expansion is performed using Legendre polynomial basis functions, which are advantageous because they ensure numerical stability and convergence properties, especially for high-dimensional problems. Legendre polynomials are defined on the interval $[-1, 1]$, this means the necessity to scale and shift the Legendre polynomials to fit the input variable intervals, which involves mapping each variable from its original interval $[a_i, b_i]$ to obtain the scaled version ρ_i of X_i into $[-1, 1]$ using the following transformation (16).

$$\rho_i = \frac{2(X_i - a_i)}{b_i - a_i} - 1 \tag{16}$$

Therefore, the mathematical formulation of the problem is given as follows (17).

$$T = \sum_{\alpha_i}^{N_1} \sum_{\alpha_2}^{N_2} .. \sum_{\alpha_7}^{N_7} \alpha_{\alpha_1 \alpha_2 .. \alpha_7} P_{\alpha_1}(\rho_i) P_{i_2}(\rho_2) .. P_{i_7}(\rho_7) with : P_{\alpha_i}(\rho_i) = \frac{1}{2^{\alpha_i} \alpha_i !} \frac{d^{\alpha_i}}{d \rho_i^{\alpha_i}} [(\rho_i - 1)^{\alpha_i}] \tag{17}$$

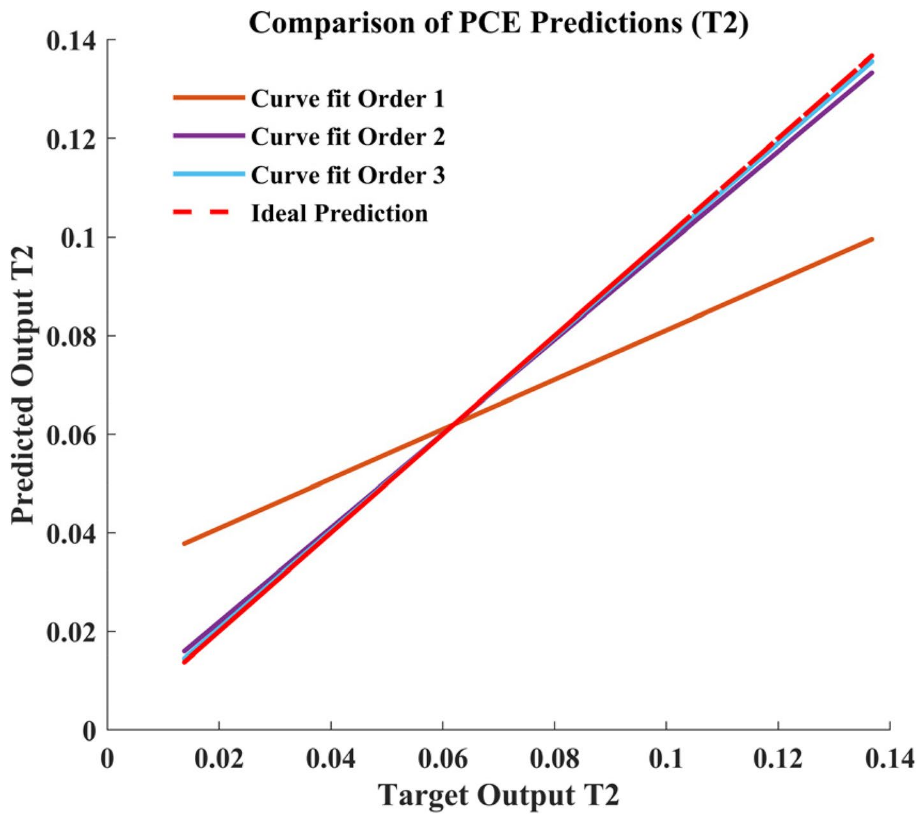

Fig. 10 PCE Orders comparison for T2

Where:	
$\alpha_{\alpha_1\alpha_2..\alpha_7}$	The coefficients of the expansion
$N_1, N_2, \ldots \& N_7$	The maximum degrees for the Legendre polynomials for each input variable respectively
$\alpha_1, \alpha_2, \ldots \& \alpha_7$	The degrees of the Legendre polynomials for each input variable
N_k	The orders of the expansions for each input parameter, indicating the highest degree of polynomial terms included in the expansions
$P_{\alpha_i}(\rho_i)$	The Legendre polynomial of degree α_i for the ith variable X_i with the scaled value ρ_i

3.4 Finite element modeling and neural networks

The finite element modeling carried out in this part of the study is for a modal analysis of the studied reinforced concrete bridge. The objective is to use finite element analysis to solve the eigenvalues problem to define the fundamental natural pulse, frequency, and period of the bridge vibration, and consequently the fundamental modal deformed shape as displayed in Fig. 13. In this part of the study, the finite element model of the structural bridge system was developed using SAP 2000 software. The model dimensions were based on the actual geometric specifications as viewed in the profiles of Figs. 4 and 5, with the overall structure measuring 18 m in each span's length, 10 m in width, and 1.2 m in beam's height. The structural components were modeled using two-dimensional shell elements (Shell-Thin) and three-dimensional frame elements (Frame), which are suitable for capturing the behavior of both deck slabs and girder elements. A mesh convergence study was

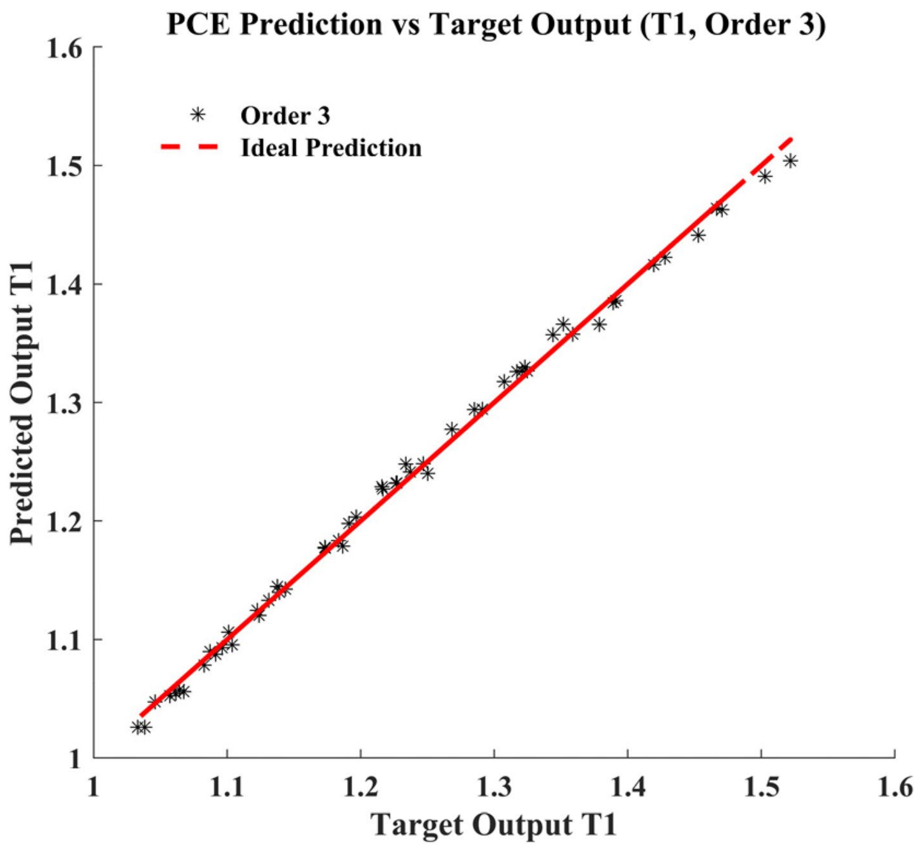

Fig. 11 PCE Regression for prediction of T1

conducted to determine the optimal element size, resulting in a mesh size of 1 m for the deck and girder elements. Boundary conditions were set to simulate realistic constraints, with fixed supports at the abutments and roller supports at the piers to allow for thermal expansion. Material properties were assigned based on standard concrete and steel specifications, with concrete having a Young's modulus of 33 GPa and a Poisson's ratio of 0.2, with a compressive strength of 30 MPa, while steel had a Young's modulus of 200 GPa and a Poisson's ratio of 0.3.

As well, the neural network approximation is implemented on the bridges data set. The learning process is based on the Levenberg–Marquardt algorithm (Levenberg 1944), (Marquardt 1963) with feed-forward back propagation using the RELU activation function. The adopted neural network architecture, as illustrated in Fig.14, has seven inputs with 2 hidden layers, considering 15 and 10 nodes for each, respectively, and one output layer (7–15-10–1). The mathematical formulation for each hidden layer node and for the output could be written as given in the following expression (18):

$$HN_k^j = max\left(0; \sum_{i=1}^{n} w_k^{j-1} HN_i^{j-1} + b_{j-1;j}\right) \tag{18}$$

where k is the node's index, j is the index number of the hidden layer, n is the total number of nodes. HN_k^j is the hidden layer j with node k, w_k^{j-1} represents the

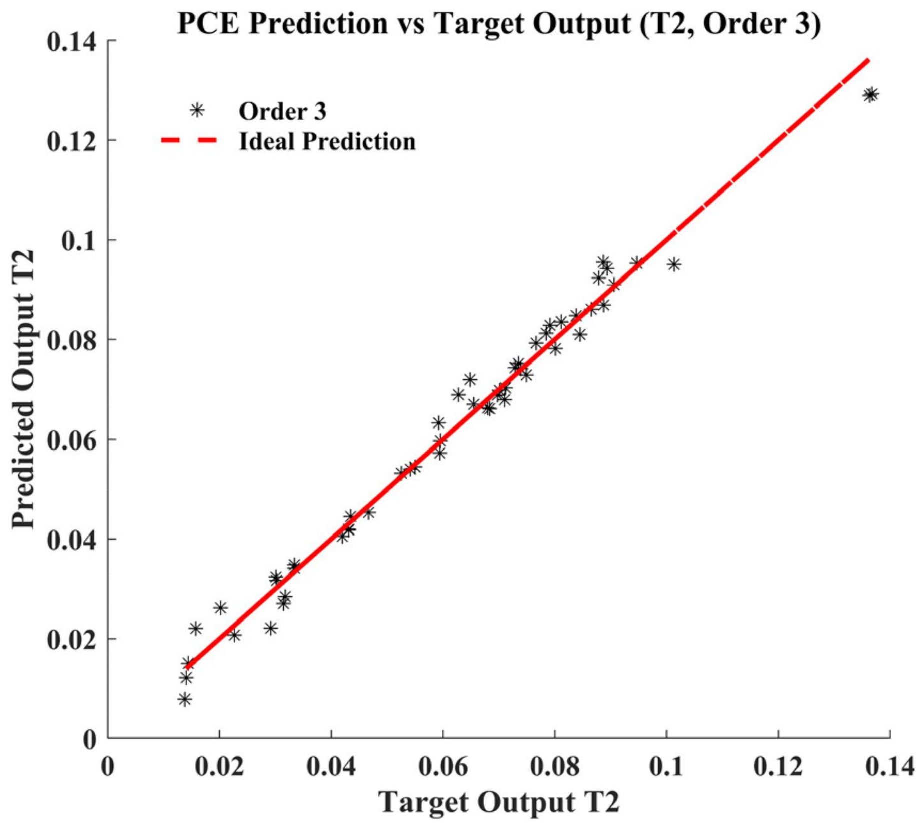

Fig. 12 PCE Regression for prediction of T2

Table 3 Inputs and PCE outputs of the Case Study

n_c	I_c	H_c	M_2	M_1	K_a	K_p	T_1 (s)	T_2 (s)
3	0.049	7.5	1876	14.81	18,981	113,065.8	1.2235	0.0981

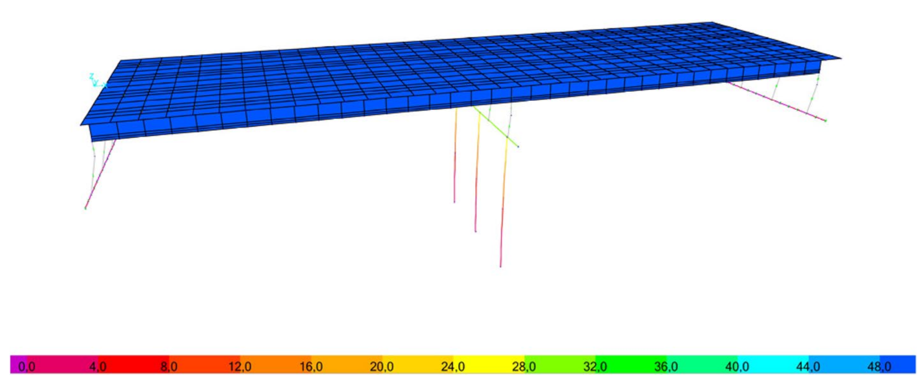

Fig. 13 Longitudinal Fundamental Deformed Modal Shape (Mode 1)

connection weight of node k between layer $j-1$ to j. While $b_{j-1;j}$ is the bias of the hidden layer j. The learning rate is fixed at 0.001. The training performance is described by the mean square error with a value of 2.42e-17, as given in the graph in

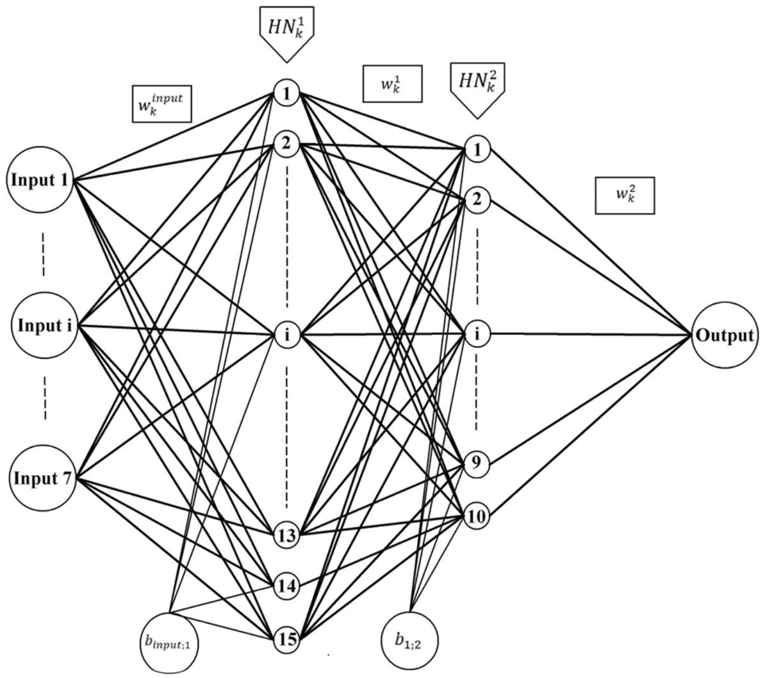

Fig. 14 Neural Networks Architecture

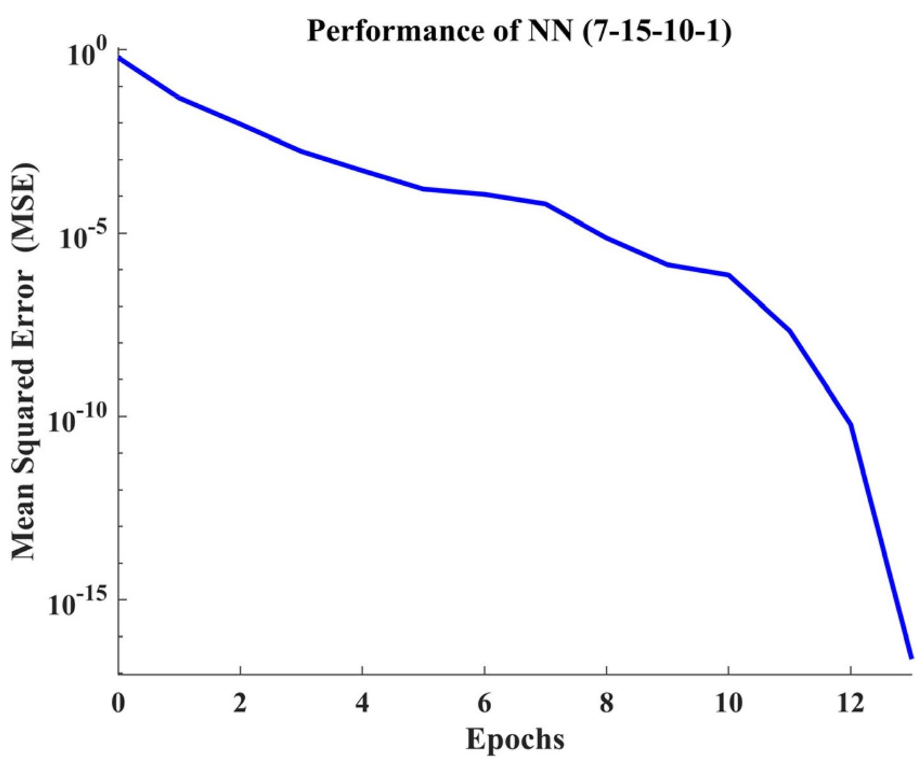

Fig. 15 Neural Networks training performance MSE

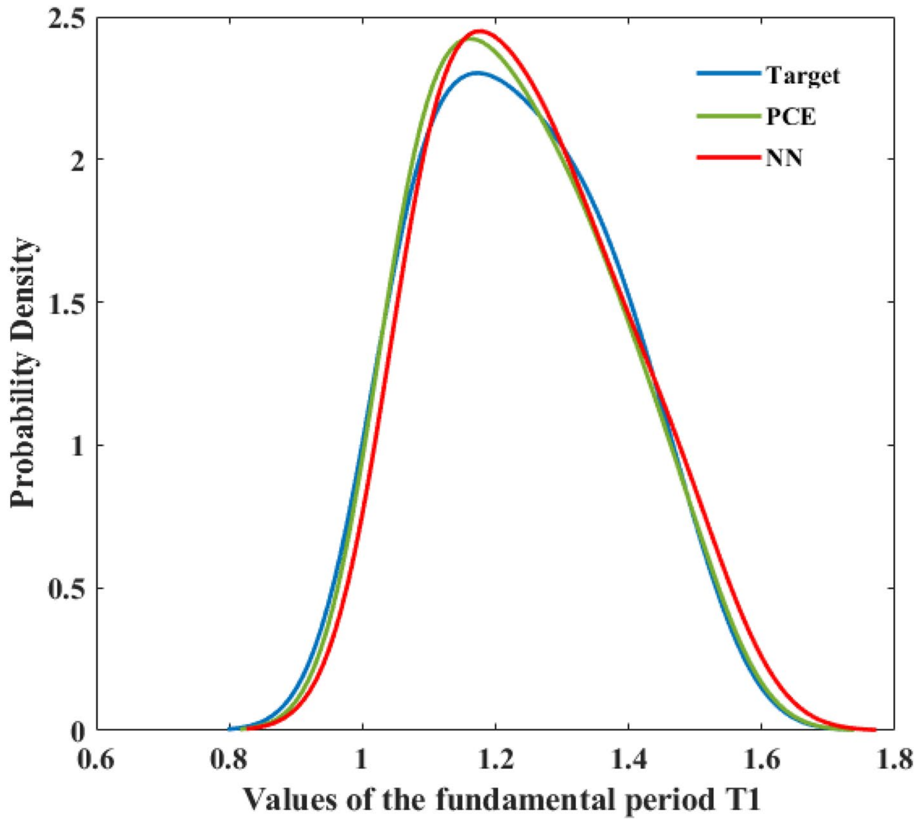

Fig. 16 Probability distributions of PCE and NN trained data

Table 4 Values of longitudinal fundamental period

Method of calculation	*T*(s)	REP %
Polynomial Chaos Expansion T^{PCE}	1.22	4.27
Finite Element Analysis T^{FEM}	0.91	22.23
Artificial Neural Networks T^{NN}	1.24	6
Algebraic Modal Eigenvalues calculation T^{Mod}	1.17	-

Fig. 15 with a descending evolution. Then the resulting regression coefficient is equal to 99.7%.

The obtained values are compared to the results of the proposed polynomial chaos model. Fig. 16 presents the comparison in terms of probability distributions. Thus, Table 4 presents the resulting values obtained using different methods and their corresponding error percentages relative to the reference value which is the algebraic modal eigenvalue. The relative error percentage (REP) is computed following the formula (19), where T^i is the approximated value of natural period with another method and T^{Mod} is the reference value obtained from modal calculation:

$$REP = \left| \frac{T^i - T^{Mod}}{T^{Mod}} \right| \times 100\% \tag{19}$$

3.5 Reliability analysis based response spectral method

The dynamic amplification factor is obtained from the elastic response spectrum of the EUROCODE 8 standard (EUROCODE8 1998) while considering the acceleration zoning as seen in Fig. 17provided in the Moroccan project guide for bridge seismic (DRCR 2009). The soil classification is considered according to the Moroccan earthquake code (RPS 2011). The elastic response spectrum is given as follows (20).

$$\begin{cases} 0 \le T \le T_B \ : \ S_R(T) = PGA.S\left[\frac{2}{3} + \frac{T}{T_B}\left(\frac{2,5}{q} - \frac{2}{3}\right)\right] \\ T_B \le T \le T_c \ : \ S_R(T) = PGA.\frac{2,5}{q}S \\ T_c \le T \le T_D \ : \ S_R(T) = PGA.\frac{2,5}{q}\,S\left[\frac{T_C}{T}\right] \\ T_D \le T \le 4s \ : \ S_R(T) = PGA.\frac{2,5}{q}\,S\left[\frac{T_D T_C}{T^2}\right] \end{cases} \quad (20)$$

The corresponding studied structure is situated in Oujda City, and the corresponding seismic zone is M4 with a mean peak ground acceleration of 1.35 m/s^2. In addition,

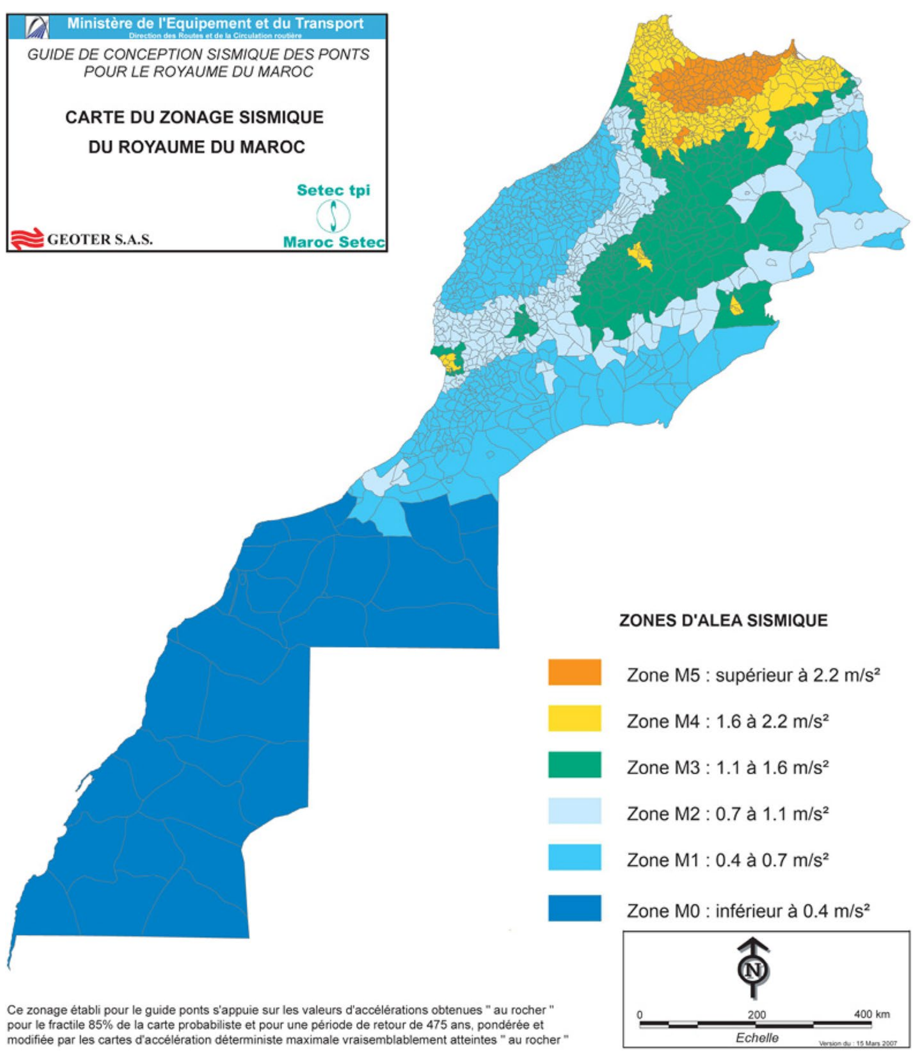

Fig. 17 Peak Ground Acceleration Seismic Zoning Map of Morocco (DRCR 2009)

based on geotechnical investigations, the support soil is considered to be very dense soil and soft rock, which is a soil of class 1 in accordance to the Moroccan seismic regulations code (RPS 2011) and by equivalence with the European code (EUROCODE8 1998), it is a soil of type B. Therefore, the soil coefficient is equal to 1.35. The predicted fundamental natural period is equal to 1.221. The justification rule is considered not to be respected when the dynamic stress exceeds the allowable concrete stressσ_c. In this situation, the failure probability is given by $P_f = P_{rob}(\sigma_{dyn} > \sigma_c)$. Then, the limit state for the seismic stress violation is formulated as follows (21).

$$G = 0.6f_{c28} - \left[\frac{32}{(1.316D)^3\pi}M_d.\frac{2,5}{q}S\left(\frac{T_D T_C}{\left(T^{PCE}\right)^2}\right)PGA\right] \tag{21}$$

Where:	
f_{c28}	28th day concrete compressive strength
M_d	Mass of the bridge deck
D	Pier diameter
T^{PCE}	PCE predicted fundamental natural period
T_C&T_D	Spectral periods
PGA	Peak Ground Acceleration
S	Soil coefficient equal to 1.2
q	Behaviour factor equal to 1.5

Reliability analysis for the corresponding failure criterion is conducted directly using the Monte Carlo simulation for 10e7 iterations. The statistical parameters of the problem variables are outlined in Table 5, and the resulting convergence of the probability of failure is depicted in the graph of Fig. 18. Fig. 19 shows Monte Carlo simulation samples, while Table 6 Presents the numerical results of the analysis. It is noteworthy that UQlab framework for uncertainty quantification (S. Marelli 2024) is used to perform reliability analysis with high efficiency and ease.

4 Discussion

The significance of the study is demonstrated by the precision of the results obtained. Polynomial chaos expansion, in parallel to neural networks, presents an efficient way to get an idea of structure-dynamic behavior by simplifying the calculation of the natural vibration properties of reinforced concrete bridges. As remarked from the results,

Table 5 Statistical parameters of variables

Var	Unit	PDF	Nom	Mean	Std
f_{c28}	MPa	Log	35	35	7
PGA	m/s^2	Norm	1.35	1.35	0.25
M_d	T	Determ	1876	-	-
T^{PCE}	s	Determ	1.2235	-	-
T_C	s	Determ	0.25	-	-
T_D	s	Determ	1.2	-	-
D	mm	Determ	1000	-	-

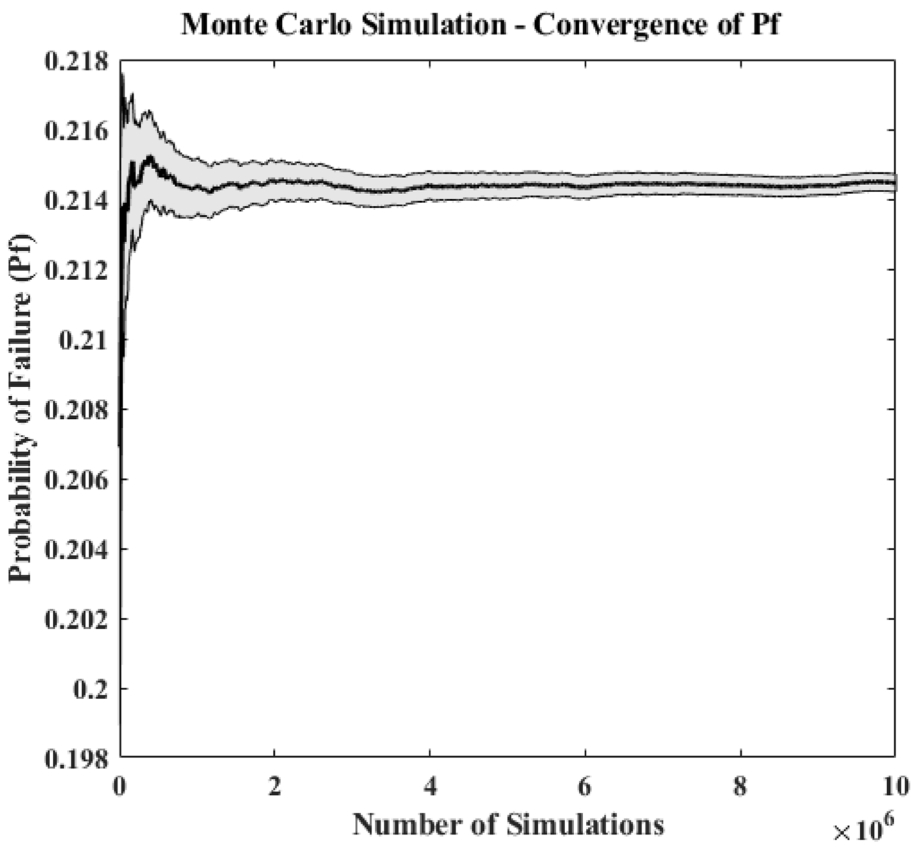

Fig. 18 Monte Carlo simulation—Probability of failure convergence

the polynomial development is conditioned by the adequate choice of the polynomial order; the findings are more precise to an acceptable regression proportionally with a high order of three that led to a minimized mean square error with a value of 6.0144e-05 and a regression coefficient of 99%. In addition, the influence of the basis functions is important.

In the study, Lagrange basis functions are adopted instead of the monomial and the other complex basis functions like Jacobi and Hermite, etc., for two principal reasons: good approximation with less computation cost. In fact, monomial is considered the simplest polynomial form, and it is not very competitive, especially when dealing with multiple inputs. On the other hand, complex polynomial functions present the drawbacks of the high calculation cost considering the timing and especially the necessity to scale every variable to their definition domain. Accordingly, the Lagrange polynomial is adopted as an intermediate solution due to its numerical stability and good regression for multidimensional problems. The only hindrance was shifting the Legendre polynomials to fit the input variables into their definition domain. Hence, the results demonstrate clearly the effectiveness of polynomial chaos metamodeling for predicting eigenvalue properties that characterize the dynamic behavior of reinforced concrete bridges.

Moreover, the predicted value of the fundamental period is compared directly with the resulting values obtained through finite element modeling and artificial neural network, as well to the analytical eigenvalue resolution. The polynomial chaos expansion method

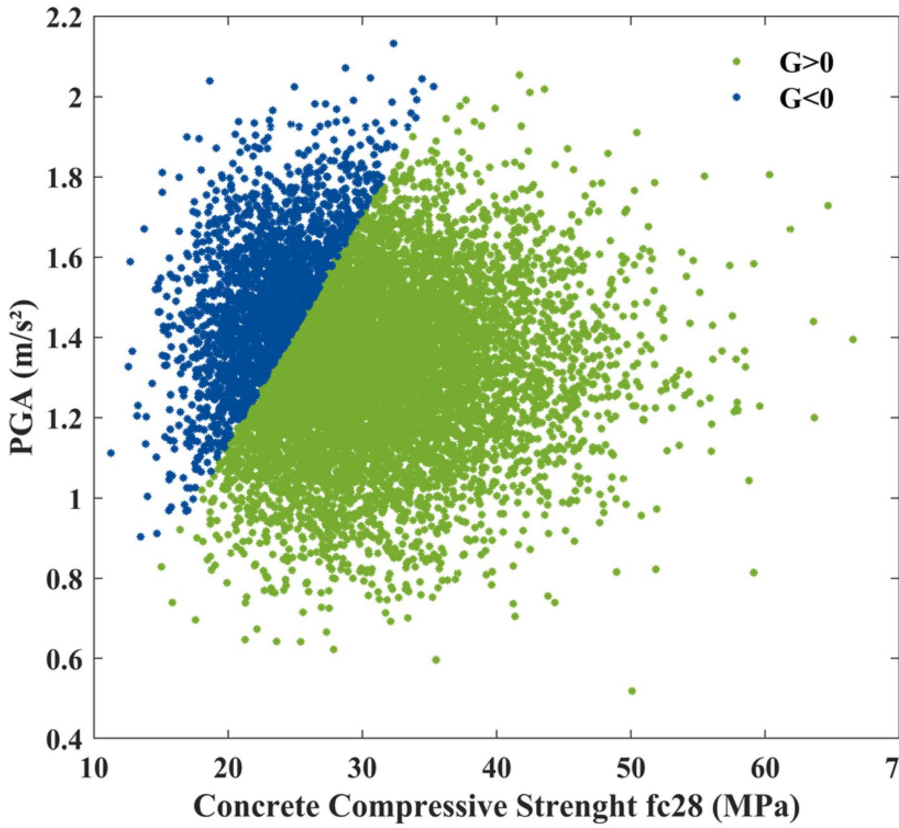

Fig. 19 Monte Carlo simulation – Samples

Table 6 Monte Carlo simulation—Reliability Analysis results

Reliability index	Probability of failure (%)
0.8	21.4

provides a fundamental period of 1.22 s, resulting in an error of 4.27%. This small deviation indicates that the method offers a relatively accurate prediction of the fundamental period. Similarly, the artificial neural networks method produces a period of 1.24 s with a 6% error, also reflecting a high degree of accuracy. In contrast, the finite element modal analysis gives a fundamental period of 0.91 s, which corresponds to an error of 22.23%. This higher error relatively could be attributed to several factors. One possible reason is the sensitivity of the finite element model to the mesh quality and element type modeling. Additionally, the simplifications made in the finite element model, such as boundary conditions and material characteristics considering damping properties, can also influence the accuracy of the predicted fundamental period. Overall, the results are very comparable and present a more nuanced understanding of the practical utility of surrogate modeling. Furthermore, ensuing the proposed methodology, the study extends to enhance the implementation of Weiner expansion in conjunction with Monte Carlo simulation for structural reliability of bridges, considering the seismic stress failure criterion, stepping the predicted fundamental period of the bridge vibration, and performing response spectrum analysis to define the corresponding limit state function. The reliability index converges to a value of 0.8, and the failure probability is approximately 21.4%. As an interpretation, it can be acceptable to admit the actual dimensioning of the pier system, but it does not ensure its functionality through the service life of the bridge within a safety domain because, in the major cases, it is desirable to have a probability of failure less than 10%. It is important to outline that the meaning of failure in this study does not mean collapse, but merely the violation of the stress justification criterion according to the normative code for the limit state function.

The strengths of the model lie in its ability to accurately predict dynamic vibration properties while minimizing computational costs. The model's high precision, evidenced by the minimized mean square error, underscores its reliability in practical applications. Furthermore, the integration of the polynomial chaos method with Monte Carlo simulation in the context of dynamic analysis enhances reliability analysis, ensuring compliance with seismic design standards and safety criteria. Our study's advantages are highlighted by its precision and efficiency in predicting natural vibration properties, offering significant advancements over traditional methods. While initially applied to reinforced concrete bridges, the methodology's adaptability to other bridge types such as prestressed concrete bridges and steel bridges is feasible with adequate dynamic model definition and adjustments in data's parameters considering the material properties and geometrical characteristics of bridge components. This flexibility is supported by the statistical basis of polynomial chaos, minimizing dependence on bridge-specific details. While our results provide valuable insights into the use of the polynomial chaos technique, and although it is widely accepted, it is important to acknowledge certain limitations described by the necessity of a large amount of design data for higher approximation accuracy. Which also depends potentially on the increase in polynomial degree, making the process time-consuming to achieve a minimized mean square error and an acceptable fit for the study's data.

5 Conclusion

The study presents a more intricate perspective on the practical use of polynomial chaos expansion to describe the dynamic behavior of reinforced concrete bridges characterized by predicting their natural vibration properties. Then the following conclusions can be drawn from this study:

- Efficient Eigenvalue Estimation: The adoption of metamodeling, particularly polynomial chaos expansion, proves to be a highly efficient alternative for estimating eigenvalues compared to traditional methods such as solving determinants of motion matrix systems algebraically. This approach offers a more robust and computationally efficient way to predict the natural vibration properties of reinforced concrete bridges.
- Reliability Analysis Enhancement: By integrating polynomial chaos with Monte Carlo simulation, the study demonstrates its utility in reliability-based response spectrum analysis. This method provides a rigorous framework for interpreting seismic design standards, ensuring safety through dynamic failure criteria. Such advancements are

necessary to improving the resilience of bridge structures under varying environmental conditions.

It will be important for future work to consider the bridge as a multiple degree-of-freedom structure to improve precision in vibration analysis. This approach can anticipate eigenvalue calculations without the need for solving robust matrix systems, which are often expensive and time-consuming. Additionally, future perspectives should account for Dynamic Soil-Structure Interaction (DSSI), as it significantly influences the dynamic behavior of bridge structures.

Finally, this study contributes significantly to the field of structural engineering and reliability by advancing the application of polynomial chaos expansion offering valuable viewpoints for both researchers and structural engineering practitioners.

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Authors' contributions
H.L.: Conceptualization, Methodology, Investigation, Analysis, Writing – Original draft. M.E. and N.L.: Supervision, Writing review and editing.

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Availability of data and materials
The data used to support the findings of this study are available from the corresponding author upon request.

Declarations

Competing interest
The authors declare that they have no competing interests.

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