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The uncorroded-part fallacy

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Abstract

This paper deals with the collapse load of corroded parallel steel units, such as sets of corroded strands alone, or embedded in a concrete core. It is shown that what is relevant is the distribution of the damage between the units, and not the total area loss due to corrosion. The paper also shows that assuming that the area loss is related to the limit load loss is misleading, and potentially dangerous.

Keywords: Corrosion, Strands, Pitting, Preloaded concrete, Polcevera Viaduct, Collapse, Progressive rupture, Progressive collapse, Bridge retrofitting

1 Introduction

Corrosion is a very important and frequent cause of failure of engineering structures. A particular problem arises when key resisting members are totally or partially made by a set of steel units working in parallel under high tensile forces and attacked by corrosion.

This kind of system is widely used, e.g., a set of steel strands or single wires that are initially identical are coupled to carry a high tensile force: preloaded concrete, steel stayed bridges, earth reinforcing, concrete dams, ziplines, etc.

The recent collapse of the Polcevera Viaduct, which happened on August 14, 2018 and resulted in 43 deaths, many injuries, and enormous economic costs, has shown that the reasoning adopted by a good part of the engineers, before and after the collapse, is rooted in a dangerous fallacy that in-depth analysis shows to be totally groundless, similar to all fallacies.

This fallacy apparently seems a reasonable and agreeable way of thinking, but when this is inspected in detail, it appears to be completely wrong. The core of the fallacy is assuming an amount of global area loss which seems to be on the safe side, i.e. an overestimate. This area loss is then assumed to be hitting a limited number of units (a percentage of units equal to the total percentage of area loss) concentrating the overall area loss only in those units, while all the others remain implicitly *virgin*. The resisting load is then evaluated summing up the resistances of all the virgin units, all equal. This may lead to significant unsafe overestimates of the load carrying capacity.

Indeed, what may happen is that quite many units are hit, all differently, so leading to an ordering of the resistances that were once all equal. The global area loss due to corrosion may be quite low, but the loss of capacity of the system dramatic.

As systems relying on strands under tensile forces and possibly subjected to corrosion are quite numerous all around the world, it seems necessary, in order to protect lives, to explain why the fallacy is wrong, and how structural engineers should instead reason.

The system that will be analyzed in this work is a set of n 7-wire strands or instead n single wires (or *units*) acting in parallel to carry a high tensile force. n is a discrete number such as 12, 16, 32 or much higher, as in the collapse of the Polcevera Viaduct, where n was equal to 464. It is assumed that the strands were initially all equal and uncorroded. The following structural configurations will be analyzed in detail, describing the quantitative rules that explain the complete failure of the set due to ongoing corrosion starting at time t_0:

1. (Type 1). The units act alone, i.e., with no collaboration of concrete. The units together form a structural element that is loaded by a global tensile force N_0, which is assumed to be constant during the life of the element itself. Initially, each strand is loaded by a force $F_0 = N_0/n$, which implies a utilization equal to f_0 of each strand, $f_0 = F_0/R_0$, where R_0 is the original uncorroded resistance of each unit. Starting from time $t = t_0$, corrosion attacks the units in one or more sections. The problem is to assess when, i.e., for which level of corrosion, the element will fail, i.e., when all the units will break.
2. (Type 2). The units are in a concrete core, to which they are bonded, and the binary (steel strands + concrete) structural element is initially under pure external tension: the concrete is initially compressed by the action of the strands and then pulled by a part of an externally applied tensile force N_0, which reduces the compression initially applied by the strands; the strands are under a tensile force to assure preload, which is further increased by the part of the externally applied tensile load N_0 that is not taken by concrete decompression. The resulting tensile force in each of the units is initially a fraction equal to f_0 ($0 < f_0 < 1$) of their uncorroded resistance. The resulting compressive stress (negative) in the concrete is σ_0. The binary structural element (concrete + steel strands) is under an external tensile force N_0, which is assumed to be constant during the life of the element. Again, the problem is to assess when the binary element will fail, i.e., when it will break completely due to ongoing corrosion.
3. (Type 3). The units are in a concrete core and bonded to it, and the binary structural element is under a bending moment M_0 and possibly an axial force N_0 in the section under examination. These loads are assumed to be constant during the life of the structural element. The concrete core is initially stressed by (a linear distribution of) compressive stresses, while the units are under a tensile force. It will be assumed that the position of the units in the core is such that the force taken by each unit can be assumed with low error equal to that of the others, i.e., the strands are very near one another. Initially, all the units are under a force F_0, and their nondimensional load is $f_0 = F_0/R_0$. Initially, the bottom extreme fiber of the concrete is under a compressive stress σ_{bot0}, while the topmost extreme fiber of the concrete core is under a compressive stress σ_{top0}. In ordinary bridges, statically determinate, the higher compressive stress is at the top, the lower at the bottom fiber, after dead and live loads are fully applied to get the externally applied loads M_0 and N_0.

Usually, the corrosion damage of the units is not known with precision when the structural element is in working condition. The effects of corrosion are frequently described by the simplifying rule "a percentage x of the steel area is lost".

As will be seen in the next sections, this is not meaningful and may lead to catastrophic errors.

What is relevant to the estimation of the load carrying capacity of the structural elements (of type 1, 2 or 3) is the *distribution* of the corrosion of the units between themselves, not the overall area loss.

The distribution of the corrosion damage may indeed be estimated by simple rules that are on the safe side.

2 Initial remarks

2.1 The distribution of the damage in real-world resisting units

Usually, units have a significant length and transfer tensile force from one point to another.

As has been shown many times starting at least from the Ynys y Gwas collapse (Woodward 1988a, 1988b), corrosion typically does not hit the resisting units along all their lengths. More frequently, and dangerously, pitting corrosion attacks very limited portions of the unit.

If the unit is a 7-wire strand, the central wire (so-called "king") is usually undamaged, while part of, or all the outer wires are hit by pitting.

Corrosion may attack single sections, more or less severely damaging one or more wires. Along the length of the unit, these corroded sections are usually a limited part of the overall length.

It is extremely difficult to properly model the exact distribution of corrosion or to know it, so engineering methods are needed to reliably assess the load carrying capacity.

2.2 The invariance of unit stiffness while corrosion spreads

The global stiffness of a single long unit in working conditions, and attacked by corrosion remains almost unchanged.

Let $L_c = kL$ be the overall length of the unit where corrosion is found (also at different sections), with k ranging say from 1/10 to 1/1000. Let $A_r = hA_{s0}$ be the *minimum* remaining area along the length of the unit, i.e., $A_r = \min\{A_{s0}\text{-}d(x)A_{s0})\}$, where x is the position along the unit and $d(x)$ is the damage, i.e., the fraction of the total initial area A_{s0} lost due to corrosion: if the area loss is A_c, $d(x) = A_c(x)/A_{s0}$. As we take the minimum remaining area, we overestimate the elongation and *thus underestimate* the stiffness of the corroded unit. The practical value of h may range from 0.99 to 0.5–0.7. Lower values (remaining area less than 50%) are clearly situations out-of-control.

The stiffness of the uncorroded unit is $K = EA_{s0}/L$, where E is Young's modulus. The stiffness K_c of the corroded unit is:

$$K_c = \frac{1}{\int_0^{kL} \frac{dx}{EhA_{s0}} + \int_0^{(1-kL)} \frac{dx}{EA_{s0}}} = \frac{EhA_{s0}}{kL + (1-k)Lh}$$

The ratio of the corroded stiffness to the uncorroded stiffness, η, is:

$$\eta \equiv \frac{K_c}{K} = \frac{h}{k + (1-k)\cdot h}$$

With $k = 0.01$ and $h = 0.9$, we obtain $\eta = 0.999$. If $k = 1/1000$, $\eta = \sim 1$. With $k = 0.1$ and $h = 0.7$, we obtain $\eta = 0.959$. Therefore, the global stiffness of the corroded units remains almost unchanged in most of the relevant cases.

This is confirmed by experimental results for strands (Chi-Ho et al. 2019, Jeon et al. 2017, Jeon et al. 2019, Jeon et al. 2020, Lee et al. 2017). In these studies, a slight change in stiffness between the corroded and uncorroded strand specimens is observed (10–15%), but it must be pointed out that the length of the specimens L was always approximately 1 m and that they were corroded along their full extension, so a value of L much lower than that expected in units in working conditions and k near 1. For a long unit attacked by corrosion in an overall limited part of its length (as normally occurs), the change in stiffness is negligible.

This has very important consequences, as if the global stiffness of the unit remains almost unchanged, the displacements will change slightly, and so monitoring may be ineffective: the collapse is fragile. Moreover, as we shall see, there are unlucky corrosion distributions that trigger the collapse for quite low global damages.

If the stiffness of the corroded units remains almost unchanged, it is correct to assume that when the corrosion spreads, the global force is equally shared by the remaining unbroken units, regardless of their difference in corrosion extension, distribution and severity.

This implies that by increasing corrosion and lowering the resistance, the force taken by the time-corroding unit will remain practically unchanged until it breaks. It is not true that if the unit weakens, the force taken by it accordingly diminishes. The force remains unchanged, while the resistance diminishes.

The similarity between stiffnesses, however, cannot be extended to resistances, which can be quite different among the remaining units.

2.3 Effects of the sliding links between the units

The mechanical behavior of a set of parallel units while the breaking of some of them occurs is strongly influenced by the existing sliding links between them.

If a unit under tensile force breaks at a given section S, it will lose its continuity and will try to shorten. The two parts into which the break splits each broken unit will suddenly shorten, and this shortening can be more or less limited by the sliding links of the broken unit.

The sliding links between the broken unit and the other surrounding units that are still unbroken can be different.

2.3.1 Case 1: absent link

If there is no sliding link between unit A, broken, and the surrounding B, C, D,… unbroken, then the whole force F_A originally taken by unit A must be transferred to the other units B, C, D…

The whole extension in the longitudinal direction of unit A remains unloaded and inactive.

If all units A, B, C, D… initially had the same stiffness, then the force taken by the surrounding units will be increased by a factor $m/(m\text{-}1)$, where m is the number of units before the break of unit A. The increase in force will be applied to all the lengths of the remaining units.

2.3.2 Case 2: stiff link

If unit A breaks and has a stiff sliding link with the surrounding units, its shortening will imply, on the one hand, an increase in the force of the surrounding units because the sliding link will transfer the force to them; while on the other hand, by the action-reaction principle, at a given length L_r (recovery length) at both sides of the broken section, the broken unit will restore its original tensile force. So:

- Along a length L_r at each side, the broken unit will be gradually reloaded up to the original force F_A.
- Along the same length, the surrounding units will experience an increase in their force. At the break section S, the force in the surrounding units will be $m/(m\text{-}1)$ times higher than the force pre-A-break. At the distance L_r from the breaking section S, their force will return to the original prebreak value.
- The recovery length L_r depends on the stiffness of the sliding link. The higher the stiffness of the link is, the shorter L_r is. The lower the stiffness is, the longer L_r is. Therefore, weak links imply long L_r values.
- Along the length $2L_r$ the increase in the tensile force of units B, C, D… can trigger another break, as with an increased force, the damage needed to trigger new breaks in B, C, D … *is lower.*

With reference to Fig. 1, if a wire of a strand breaks, its force flows into the other wires of the same strand, regardless of the existence of surrounding concrete. This occurs

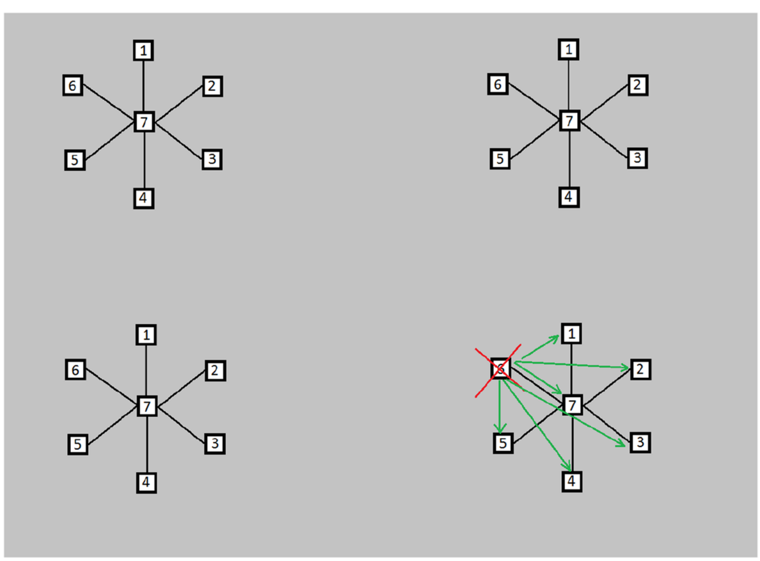

Fig. 1 Breaking of a single wire of a strand in concrete core

because the sliding stiffness of the broken wire, linking it to the siblings, is much higher than the sliding stiffness at the steel–concrete interface also if the strands are embedded in concrete, as the wires are helicoidally enveloped to the king.

When a single wire of a corroded 7-wire strand breaks (Fig. 1) its force is recovered at a given distance L_r from the breaking point, while along that length at both sides, the remaining wires of the same strand will carry the force lost by the broken wire.

Indeed, when a 7-wire strand experiences the break of one wire, this is normally one of the outer wires, as six out of seven wires surround the king (7th). Since the outer wires are helicoidally enveloped to the king, the shortening of a broken outer wire implies locking to the siblings that, thanks to friction, transfers the force lost by the wire to the other six (five, four…) (Raoof and Kraincanic 1998, Waisman et al. 2011).

2.3.3 Case 3: Sliding links with very different stiffnesses to the surrounding

Additionally, when the strand is surrounded by concrete, the force lost by the broken wire of a 7-wires strand flows into the siblings (Fig. 1) because the sliding stiffness at the concrete-steel interface is much lower than the stiffness at the steel-steel interface. The force lost by a wire flows to the other wires of the same strand and not to concrete.

This last assumption is further strengthened by the following three remarks:

1. The concrete keeps in place the broken wire, avoiding its moving away from the siblings. This helps locking due to the helicoidal envelope.
2. The bond siding link between concrete and steel is usually damaged by corrosion.
3. The bond of the strand to the concrete in posttensioned preloaded concrete structures is weakened by insufficient or imperfect injection of the ducts, which is a frequent problem in such types of constructions.

On the other hand, the *global* external stiffness of the 7-wire strand does not change much when a wire breaks because the broken wire reanchors (the reduction in stiffness is only along $2L_r$, see Sect. 2.2). Therefore, a strand attacked by corrosion in a given section is forced to take the same load until the strand as a whole breaks. The strand is *segregated* from the others and forced to carry its load until it is completely broken.

2.3.4 Case 4: sliding link with a concrete core, embedding more units

A special and quite important case of a relatively weak link is when many strands are surrounded by concrete but not directly in contact between them. In this case, the reanchoring of the broken *strand* will occur at a relatively high distance L_r, and along the length $2L_r$, the increase in force due to the loss of the just-broken strand will be shared *by the concrete and by the other strands*, proportionally to their respective stiffness (Fig. 2).

With reference to Fig. 2 strand D experiences the break of wire 6, and its force is redistributed only to the other wires of the same strand (see Fig. 1). When wires 2 and 4 also break, due to the increase in corrosion with time, the whole strand D breaks. Its force is then, only then, redistributed to concrete and other strands, proportionally to their axial stiffness. The concrete core is the medium that redistributes the lost force by shear.

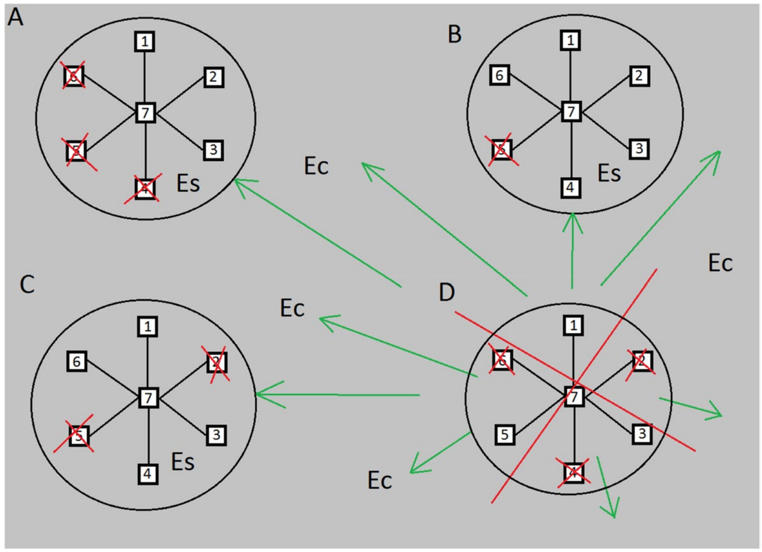

Fig. 2 Breaking of a strand in a concrete core

2.3.5 Consequences

Therefore, when corrosion spreads:

- Initially, the wires will break, and the force lost by them will be redistributed to the sibling wires *of the same strand*.
- When a strand breaks, its force is redistributed to the other strands and, if they are in a concrete core, also to the concrete, proportionally to their stiffness.
- The force taken by the other strands does not change when the *wire* of a strand breaks. It increases when that *strand* breaks, as its force must be taken by others and, if existing, by concrete.

3 Load carrying capacity of a single corroded strand

Several studies have addressed the problem of evaluating the load carrying capacity of a corroded 7-wire strand, including Pillai et al. 2014, Ebeling et al. 2016, Lee et al. 2017, Chi-Ho et al. 2019, and Jeon et al. 2020.

The fundamental result of these studies is that the load carrying capacity of a single corroded strand depends on the distribution of corrosion between the wires and cannot be evaluated by multiplying the uncorroded area of the strand by its ultimate tensile stress: the resistance loss of the strand is higher than its sectional area loss. This in particular true when pitting corrosion damages the units (Jeon et al. 2020).

In particular, two simplified methods will be recalled here for their ease of use and good prediction of the experimental results.

The first considers the area loss due to corrosion in the strand as a whole at a given section. This area loss A_{corr} is the sum of the area losses of all seven wires. The ratio between this area loss and the original uncorroded area of strand A_{s0} is by definition the damage d:

$$d = \frac{A_{corr}}{A_{s0}}$$

If the resistance of the uncorroded strand is R_0 and the resistance of the corroded strand is R, the nondimensional resistance r of a corroded strand can be written as

$$r = \frac{R}{R_0}$$

The evaluation of the resistance of the corroded strand is made by introducing a simple linear relationship between r and d in the following form:

$$r = 1 - \alpha d \tag{1}$$

where α is a parameter greater than 1, whose values have been proposed in the range 1.3–3.1 ($\alpha = 3.07$ Lee et al. 2017, $\alpha = 1.5282$ Jeon et al. 2020): in this work, the value of α assumed for some of the examples will be $\alpha = 1.5$, which is a value frequently assumed. The value of 1.3 is in good agreement with the properties of the corroded strands of the Polcevera Viaduct (Rugarli 2020b). However, all the formulations will be carried out leaving α undetermined as an available parameter. Method (1) can also be used for single wires, not strands, as was frequent in constructions built decades ago.

Therefore, the resistance loss is greater than the area loss (an identical loss would have implied $r = 1\text{-}d$, $\alpha = 1$). This keeps into account the stress intensification related to corrosion.

The second formulation proposed that will be mentioned here stems from the observation that the area loss of the most corroded *wire* of the strand is the best predictor of the resistance loss of the whole *strand* (Ebeling et al 2016, Chi-Ho 2019). If A_{0w} is the uncorroded area of a *wire* and A_{ci} is its area loss ($i = 1$–7), the area loss of the most corroded wire c_{max} can be evaluated as

$$c_{\max} = \max_{i=1-7} \left\{ \frac{A_{ci}}{A_{0w}} \right\}$$

Ebeling et al. (2016) propose the following predictor of the resistance of the corroded strand:

$$r = -0.690 c_{\max}^2 - 0.239 c_{\max} + 0.997 \tag{2}$$

This expression has been obtained by normalizing the results proposed in Ebeling et al. 2016 to express them using nondimensional parameters (Rugarli 2020a).

As the aim of this work is to obtain simple methods to assess the resistance of a set of corroded strands, the analytically simpler formulation (1) will be used.

Given a strand i, if the tensile force that it must carry is F, the limit condition is reached when the force equals the resistance, i.e., when

$$F = R$$

and normalizing to R_0, in nondimensional form when

$$f = \frac{F}{R_0} = \frac{R}{R_0} = r$$

where f is the nondimensional load level.

It is assumed that initially, when all the units are not corroded, the force that each of them must carry is F_0, so they are all loaded at $f_0 = F_0/R_0$, and $r = r_0 = 1$ ($f_0 < 1$). When the corrosion spreads, the resistance of each unit is r_i, and in general, if the unit is corroded, $r_i < r_0 = 1$. At the same time, if some units have broken, the load f is no longer f_0.

Therefore, to evaluate the ultimate condition, the problem is to compare f and r for all the units.

4 Engineering estimate of the distribution of corrosion damage between the strands

With no loss of generality, the resisting corroded units can be ordered in such a way that at time t, the first unit ($i = 1$) is the most corroded, while the last ($i = n$) is the least corroded. This means assuming a distribution of damage having the following properties:

$$d = d(i)$$
$$d_{i+1} \le d_i$$
$$\max_{i=1-n} \left\{ d_i \right\} = d_1$$
$$\min_{i=1-n} \left\{ d_i \right\} = d_n$$

$$0 \le d \le 1$$

The curve fitting the points (i, d_i) in the plane (i-d) is the distribution of damage between the units at a given time t. This curve is defined between the abscissae $i = 1$, $i = n$; it must be decreasing (i.e., $d' < 0$) and must pass through the points A(1, d_{max}) and C(n, d_{min}). The ordering of the units by decreasing damage strongly limits the shape of the curve, so it is relatively easy to assume a shape for the related function: linear, parabolic, logarithmic or exponential are immediate candidates.

The distribution may have a branch at $d = 0$, and there can be a change in slope, namely, in the point connecting a constant branch to others.

Among all the possible decreasing functions, the easiest is the linear function, i.e., in plane i-d,

$$d(i) = d_{\max} \cdot \left(1 - \frac{i-1}{i_{\lim} - 1} \right) \tag{3}$$

where d_{max} is the maximum damage read in the set (for $i = 1$), and i_{lim} is the index of the first unit with no damage at all, i.e., the first unit for which $d = 0$. If this i_{lim} is lower than n, then for higher values of i, $d = d_{min} = 0$ (heart curve, Fig. 3). If instead $i_{lim} > n$, then the least corroded strand, n^{th}, has damage

$$d_{\min} = d_{\max} \cdot \left(1 - \frac{n-1}{i_{\lim} - 1} \right)$$

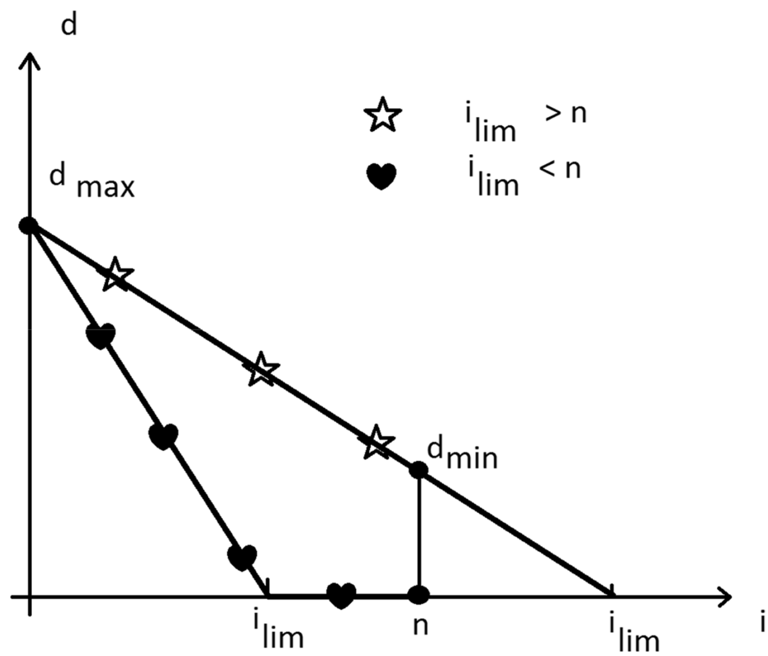

Fig. 3 Possible linear distributions of damage among the strands. In the heart curve, the damage has not hit all the units, while in the star curve, it has

that is reached when $i = n$ (star distribution, Fig. 3). It must be pointed out that i_{lim} may be higher than the number of available units. If so, it is merely a parameter useful to assess the value of d_{min} for $i = n$ while keeping a unique formal equation, (3).

Using linear distributions of damage, with reference to Fig. 4, the progressive spreading of corrosion among the strands with time can be described by the progressive shift of control points A and B, from positions A'-B' to positions A''-B'', and so on.

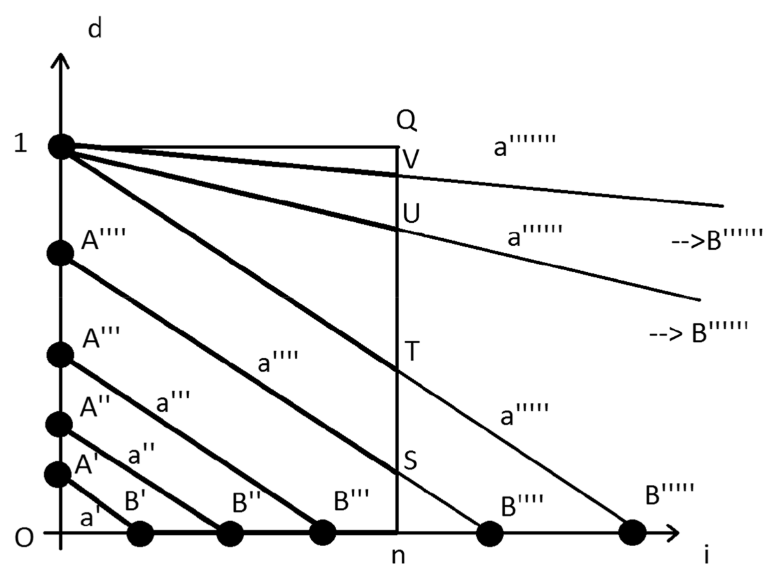

Fig. 4 Different linear distributions of corrosion at increasing time: (A'-B')-(A''-B'')-(A'''-B'''). Bolded lines A'-B'… A'''-B''', A''''-S, 1-T, 1-U, 1-V, and 1-Q are the distributions, and thin lines are the linear curves originating from them

The speed of movement along the axis d of point A is related to the speed of corrosion in the most corroded strand. The speed of movement of point B along axis n is related to the increasing number of strands attacked by corrosion with time.

Point A cannot cross the point with coordinates (1, 1), which is reached when the first strand, the most damaged, is fully corroded, while point B can move along axis "i" with no limit. When B reaches infinity ($i_{lim} = \propto$), all the strands have the same damage, d_{max}.

At infinite *time*, the corrosion spreading can be described by the curve (1,1)-(n,1), (segment "1"-Q), meaning that all the strands have been completely corroded, with the total loss of their area. Clearly, this distribution is never reached because collapse occurs much earlier.

Point A is related to the corrosion spreading in the most corroded strand; point B is related to the number of units having some corrosion, up to point (n, 0), and then, as explained, indirectly to the damage d_{min} of strand number n.

Both quantities:

- The damage of the most corroded strand, d_{max};
- The number of units with some corrosion, i_{lim}-1, or damage (> 0) of the less corroded strand, strand number n, d_{min}

are measures of the damage that can be estimated with a *relative* ease, provided some inspection or reliable nondestructive test is made. Clearly, they are much easier to assess than to assess the exact d for each i. Instead of assessing the damage for all the units, we can assess the damage of the most corroded unit, and the number of corroded units. If the distribution of damage is assumed linear, this is sufficient.

These quantities should be evaluated with reference to section S where corrosion has been found and considering a proper L_r at both sides.

The linear distribution of damage (possibly with a constant branch $d = 0$) is not only the easiest but also coherent with the assumption that corrosion initially attacks one unit and then gradually spreads to the others, while the first unit and the others experience a progression of damage; thus, both the maximum corrosion damage in unit 1 and the number of somehow corroded strands increase with time.

The linearity of the damage distribution has been confirmed during the forensic analysis related to the Polcevera Viaduct collapse, which is one of the very rare cases where the corroded wire residual diameters (minimum diameter of corroded wires) have *all* been measured for broken wires of each strand after a real collapse. So this is experimental data, and it confirms that the damage is linearly distributed (Fig. 5).

The 464 strands in the breaking section of the southeast stay of balanced system number 9 (whose detachment, according to the forensic analysis carried out by the judge's consultants (Rosati et al. 2020), triggered the collapse of balanced system 9 of the Polcevera Viaduct) had damage linearly distributed with a very good approximation ($R^2 = 0.994$). The wires did not break at the very same longitudinal section, and some limited spread in the wire-break section position was found.

In that case (Fig. 5), by measuring the uncorroded minimum diameter of all the broken *wires* (say 464*7, more than 3,000), computing the damage for each *strand* d_i ($1 \leq i \leq 464$), and plotting them in decreasing damage order (Fig. 5), it was found by the author that

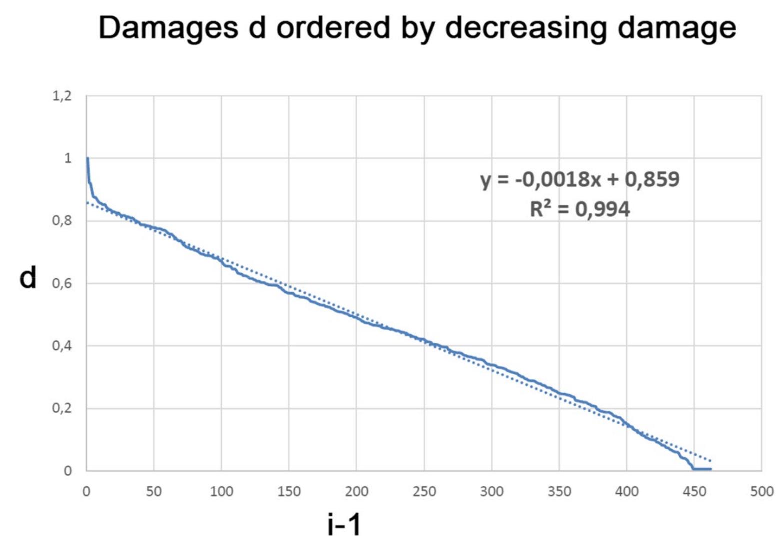

Fig. 5 The damages read in the 464 strands of the broken southeast stay of balanced system 9 of Polcevera Viaduct, ordered by decreasing damage *d*, and the related linear interpolation curve(dotted line). The damage of strand i must be read at $b=i$-1, so the curve ranges from 0 to 463 (Rugarli 2020b, with some modifications in the symbols)

the damage of the *strands* of the stay in the breaking section could be very well described by a linear fitting curve assuming $d_{max}=0.86$ $i_{lim}=478$ ($>n=464$) (for the measure of the diameters of all the broken wires Rosati et al. 2020, for the plotting of the damages d_i ordered by decreasing damage in plane *i-d* Rugarli 2020b). The percentage of total area loss can be evaluated easily from Fig. 5 by computing the trapezoidal area under the fitting curve and dividing by 463: 463* (0.859 + 0.0256)/(2*463) = 0.44, that is, 44%. The value 0.0256 is obtained by placing $x=463$ in the fitting curve $d=0.859–0.0018x$.

Some other possible distributions of damage are interesting and will be discussed with reference to Fig. 6.

The first is the distribution of damage that assumes that all the units do have the same damage and will here be named the "same damage distribution" (curve u of Fig. 6). This

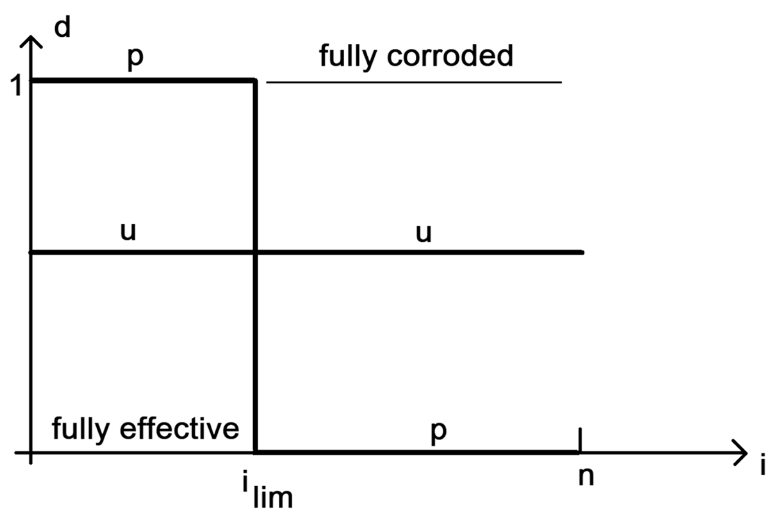

Fig. 6 The special distributions of "fully lost-virgin" (p) and "same-damage" (u) damage

assumes that all the units are exposed to the same corroding action at any time t. It could ideally be applicable to units attacked in the same way, such as those subjected to cycles of water submersion-drying. It is not realistic for a real-world set of units that are usually attacked with a geometrical pattern.

The second is the distribution that assumes that if $A_{c,tot}$ is the total area loss of the set due to corrosion, then the area loss $A_{c,tot}$ regards only some of the units that are fully corroded, while *all the others remain in a virgin state* (distribution p of Fig. 6). In this case, one can evaluate the number of units totally corroded as

$$i_{\lim} = \frac{A_{c,tot}}{nA_{s0}} \cdot n = \frac{A_{c,tot}}{A_{s0}}$$

The load carrying capacity is (the uncorroded units all have *the same resistance* R_0, which is the reason why their resistances can be meaningfully summed)

$$R_{tot} = (n - i_{\lim}) \cdot R_0 = \left(n - \frac{A_{c,tot}}{A_{s0}}\right) \cdot R_0 = nR_0 \cdot \left(\frac{nA_{0s} - A_{c,tot}}{nA_{s0}}\right)$$
$$\frac{R_{tot}}{nR_0} = \frac{nA_{s0} - A_{c,tot}}{nA_{s0}} = 1 - \frac{A_{c,tot}}{nA_{s0}}$$

which is the numerical form of the *fallacy of the uncorroded part*: the load carrying capacity of a corroded set of strands is a fraction of the original one, equal to the ratio of the areas of the uncorroded part to the original one.

Therefore, the fallacy is strictly related to the assumption that the area loss is fully concentrated in a number of strands (i_{lim}), while all the others are still uncorroded, i.e., to the assumption of the distribution of damage p of Fig. 6. Which is just not possible. What happens in reality is that the area loss will be disseminated among many different strands and that the typical corroded strand will *not* lose *all* its area before breaking. Indeed, as shown in the previous section, the area loss needed to break a strand is related to its level of load f, and it depends on the distribution of the damage between the wires. On average, assuming a unique α for all the strands

$$r_i = \frac{R_i}{R_0} = 1 - \alpha d_i$$

and

$$d_i \neq d_j$$

which is incompatible with the distribution p of Fig. 6. The corroded units will not have the same resistance: instead, there will be an ordering of their resistance, and they will break depending on their individual resistance and load.

5 Load carrying capacity of unary systems under tensile force (type 1)

5.1 Worst distribution of damage

At a given time t, let d_i be the damage of unit i, and let us assume that the units are ordered in such a way that $d_i \geq d_{i+1}$. Initially, they are all loaded at f_0.

As corrosion does not change the global stiffness of each unit, if some units are broken, the loads of the remaining units will be the same regardless of corrosion: each remaining unit will be loaded by the same amount of the others. Assuming that b strands have broken, the nondimensional load f applied to all the remaining corroded units will be equal for all the units and in particular

$$f = f_0 \cdot \frac{n}{n-b} \tag{4}$$

The *worst* possible distribution of damage between the strands is such that when b strands have broken, the damage of the first unbroken unit, whose number is $i = b + 1$, is exactly *the minimum* needed to trigger the failure of the unit $(b + 1)$ at the increased load level coherent with the breaking of the previous b broken units. This can be written in the following way, equating the load applied $f(b)$ to the residual resistance r of unit $b + 1$, as a function of the damage d_{b+1}:

$$f(b) = f_0 \cdot \frac{n}{n-b} = r\left(d_{b+1}\right) = 1 - \alpha d_{b+1}$$

That is, (b ranges from 0 to n-1)

$$d_{b+1} = \frac{1}{\alpha} \cdot \left(\frac{n-b-f_0 n}{n-b}\right)$$

Replacing $(b + 1)$ by i (ranging from *1 to n)*, we obtain the worst-of-all distribution of damage between the strands for i between 1 and n:

$$d_i = \frac{1}{\alpha} \cdot \left(\frac{n-i+1-f_0 n}{n-i+1}\right) \tag{5}$$

When b is sufficiently high, say $b = b_{lim}$, the load level f applied to the remaining strands will reach the value 1, meaning that the *uncorroded* strands will also break. This happens when

$$1 = f_0 \cdot \frac{n}{n - b_{\text{lim}}}$$

That is

$$b_{\text{lim}} = n - f_0 n$$

$i_{\text{lim}} = n - f_0 n + 1$

Therefore, the worst-of-all distribution of damage between the strands, $d^*(i)$, is such that (see Fig. 7 for a plot):

$$\begin{cases} d_i^* = \frac{1}{\alpha} \cdot \left(1 - \frac{f_0 n}{n-i+1}\right) & \text{for } i = 1 \; to \; n - f_0 n + 1 \\ d_i^* \; = \; 0 & for \; i \; \geq n \; - f_0 n + 1 \end{cases} \tag{6}$$

This distribution implies the minimum total area loss sufficient to trigger the progressive rupture of all the strands, as soon as the first strand, the most damaged, reaches the damage

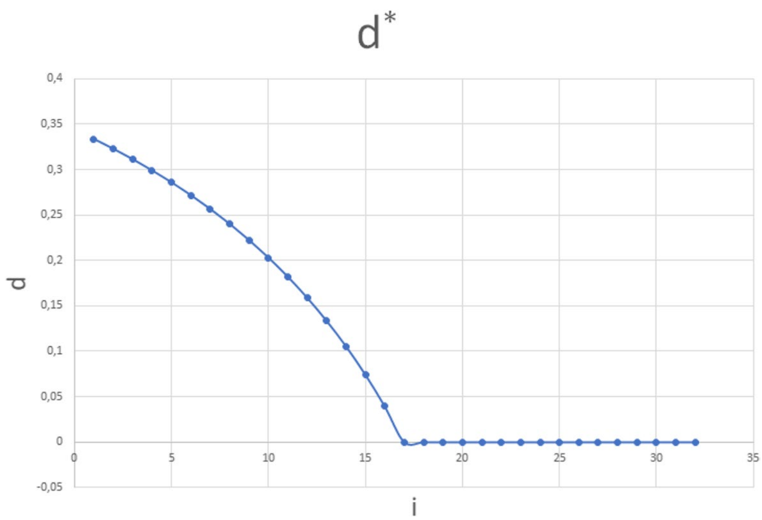

Fig. 7 Worst distribution of damage d* for $n=32$, $f_0=0.5$, and $\alpha=1.5$

$$d_1^* = \frac{1-f_0}{\alpha}$$

The next damage will be:

$$d_2^* = \frac{1}{\alpha} \cdot \left(1 - \frac{f_0 n}{n-1}\right)$$

$$d_3^* = \frac{1}{\alpha} \cdot \left(1 - \frac{f_0 n}{n-2}\right)$$

and so on. Therefore, for each unit i the damage of unit i is higher than that of unit $i+1$, and it is exactly the amount needed to carry on the rupture of it after i-1 units have broken. The damage needed to break next unit decreases with the increasing the number of broken units.

This is the worst distribution of damage, i.e., the one implying that the set of units loaded at f_0 collapses, all the units break, and the minimum overall area loss occurs.

The overall area loss related to this distribution can be evaluated by summing the damages, while the relative loss ξ, i.e., the fraction of the initial total area that is lost due to corrosion, can be evaluated by dividing by n the previous sum of the damages. So:

$$\begin{aligned}
&\xi = \frac{1}{n}\int_1^n d^*(i)di = \frac{1}{n}\cdot\frac{1}{\alpha}\cdot\left(\int\limits_1^{n-f_0 n+1}\left(1-\frac{f_0 n}{n-i+1}\right)di + \int_{n-f_0 n+1}^{n} 0\cdot di\right) = \\
&\frac{1}{n\alpha}\cdot\left(n-f_0 n\right) + \frac{1}{n\alpha}\cdot\left[f_0 n\cdot\ln(n-i+1)\right]_1^{n-f_0 n+1} = \\
&\frac{1}{n\alpha}\cdot\left(n-f_0 n + f_0 n\cdot\ln\left(f_0 n\right) - f_0 n\ln(n)\right) \Rightarrow \\
&\xi = \frac{1}{\alpha}\cdot\left(1-f_0+f_0\ln f_0\right)
\end{aligned} \tag{7}$$

Therefore, the percentage of total area loss for the worst distribution of damage depends only on f_0 and α. Table 1 summarizes the results for $\alpha = 1.5$ and different initial load levels f_0.

For instance, for load level $f_0 = 0.5$, the worst-of-all distribution of damage that triggers the collapse of the whole set implies a percentage of total area loss equal to $\xi = 10.2\%$.

In evaluating ξ, the curve d*(i) is assumed to be continuous. More precisely, if this assumption does not hold true, the ξ value can be evaluated by the following expression:

$$\xi = \frac{\sum_{1}^{n} d_i^*}{n}$$

However, if n is sufficiently high, the difference between the integral and the summation is low. For instance, for $n = 32$ and $f_0 = 0.5$, the continuous curve gives $\xi = 0.102$, while the discrete summation gives $\xi = 0.107$.

According to the fallacy of the uncorroded part, the total corroded area loss needed to trigger the collapse should have been 50%.

Therefore, it is not true that to take a set loaded at $f_0 = 0.5$ to the collapse, the percentage of total corroded area loss should be 50%. The worst-of-all distribution of damage implies a total corroded area loss of only 10.2%.

Real-world distributions of damage are not the worst but imply an area loss much lower than that assumed by the fallacy of the uncorroded part, as will be seen.

5.2 Failure condition for real distributions of damage

The failure of the corroded set occurs when for all i

$$d_i \geq d_i^*$$

With reference to Figs. 8 and 9, where the example n = 32, $f_0 = 0.5$ $\alpha = 1.5$ is plotted:

1. The distribution A'-B' (Fig. 8) is safe because for all i, $d_i <$ d*(i). The first strand is not broken because its damage is lower than (1-f_0)/α = 0.333.
2. The distribution A''-B'' is safe (Fig. 8). The first 16 strands (up to the intersecting point I) have a damage lower than that of the curve d*, so they do not break. It is then irrelevant that for $i > 16$, the damage is higher than d*(i) because the previous strands have not broken; therefore, the load level for all the strands is still f_0.
3. The distribution A'''-B''' is safe no matter some strands are broken. The first 3 strands are broken because their damage is greater than or equal to d^*. However, the damage

Table 1 Percentage of total area loss related to the worst possible distribution of damage that leads to the failure of a set of units for different initial load levels

f_0	α	ξ
0.4	1.5	0.155
0.5	1.5	0.102
0.6	1.5	0.062
0.7	1.5	0.033

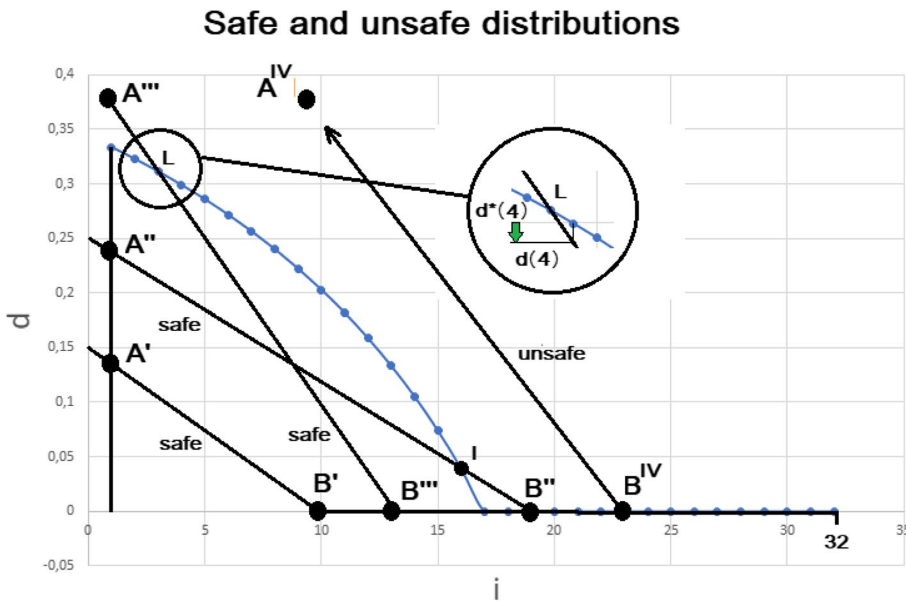

Fig. 8 Examples of safe and unsafe damage distributions. The dotted curve is the worst distribution d^*. $n=32$, $f_0=0.5$, $a=1.5$

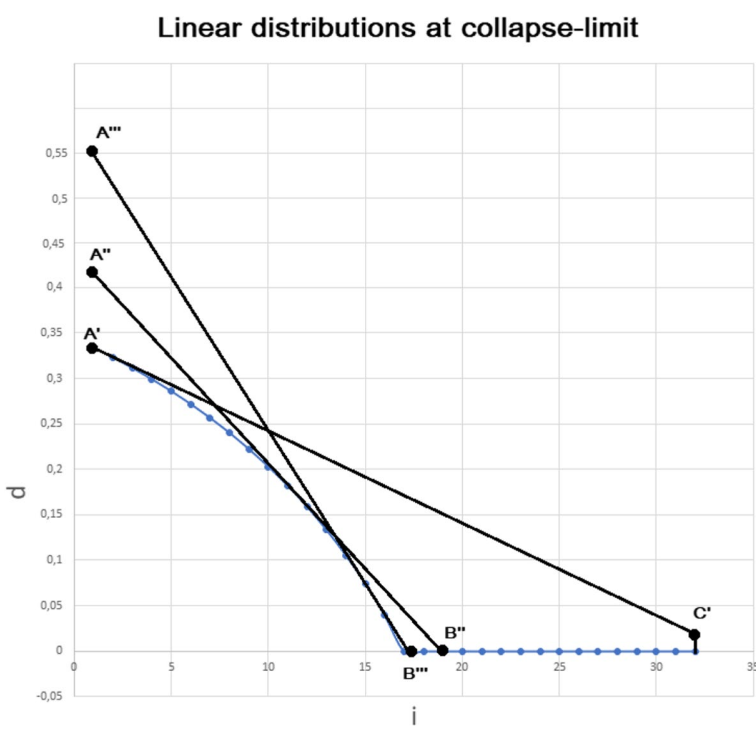

Fig. 9 Linear distributions at the limit of collapse

of the fourth strand is lower than that needed to continue the failures. After 3 strands have broken, the load level is 0.5(32/29) = 0.551. By Eq. (1), the damage needed to break strand 4 is d* = (1–0.551)/1.5 = 0.299. However, the damage *d* of strand 4 is lower than that value (see circled detail), so strand 4 does not break. This stops the failure process until the corrosion rises so that strand 4 reaches a damage of 0.299.

4. The distribution A^{IV}-B^{IV} is unsafe (Fig. 8) because all the strands have damage higher than d*(i). It is irrelevant that 10 strands are uncorroded: the load level they have to carry, *f*, is > 1, and so they also break. As 22 strands out of 32 are broken, the load level of strand number 23 is no longer $f_0 = 0.5$, but $f = 0.5(32/10) = 1.6 >> 1$. So it breaks. In addition, so on. They all break.
5. It is then clear that the limit of collapse of the whole set is reached when the real damage distribution *d*(i), assumed linear, is *tangent to the worst distribution* d*(i). Tangent distributions (Fig. 9) are then the distributions that, once reached by the ongoing corrosion, trigger an immediate complete collapse of the whole set. There are infinite linear distributions that are tangent to the worst distribution d*(i). In Fig. 9, three of them have been plotted: A'-C', A''-B'', and A'''-B'''.

Given a value of d_{max} (greater or equal to $d^*(1) = \frac{1-f_0}{\alpha}$), there is only one value of i_{lim} (Eq. 3) so that the linear distribution is tangent to the worst one. Therefore, with reference to Eq. (3), what must be found to define tangent linear distributions is a function $i_{lim} = i_{lim}(d_{max})$.

To select among all the possible linear distributions of damage in form (3) those tangent to the worst distribution d*(i), the following two conditions must be written:

1. For a certain value of *i*, the value of *d*(*i*) equals the value of *d**(i).
2. For the same value of *i* the slope of the linear distribution *d*(i) is equal to the slope (tangent) of the worst distribution *d**(i).

That is:

$$\begin{aligned} d_i^* &= \frac{1}{\alpha} \cdot \left(1 - \frac{f_0 n}{n-i+1}\right) = d_{\max} \cdot \left(1 - \frac{i-1}{i_{\lim}-1}\right) = d_i \\ d_i^{*'} &= -\frac{f_0 n}{\alpha \cdot (n-i+1)^2} = -\frac{d_{\max}}{i_{\lim}-1} = d_i' \end{aligned} \tag{8}$$

Expressing i_{lim} as a function of d_{max}, the following relationship is found:

$$i_{\lim} = 1 + \frac{2\alpha d_{\max} n}{2\alpha d_{\max} - 2 + 4f_0 + 4\sqrt{f_0^2 + \alpha d_{\max} f_0 - f_0}} \tag{9}$$

It must be underlined that these conditions are met only if the worst-damage curve is sufficiently smooth, i.e. only if b_{lim} is sufficiently high (say 8–10 or more). If the worst-damage distribution is a broken line, as happens when b_{lim} is low, the tangent condition of a linear damage distribution must be replaced by proper matching of one of the slopes of the broken line.

The point where the linear curve *d*(i) is tangent to the nonlinear smooth curve *d**(i) must not have negative *d*. If this happens, as shown in Fig. 10, the distribution *d*(i) is over the limit. Therefore, there is a range for the d_{max} values that can be related to limit conditions. By imposing that i_{lim} of Eq. (9) must be equal to *i* for which $d^*(i) = 0$, that is,

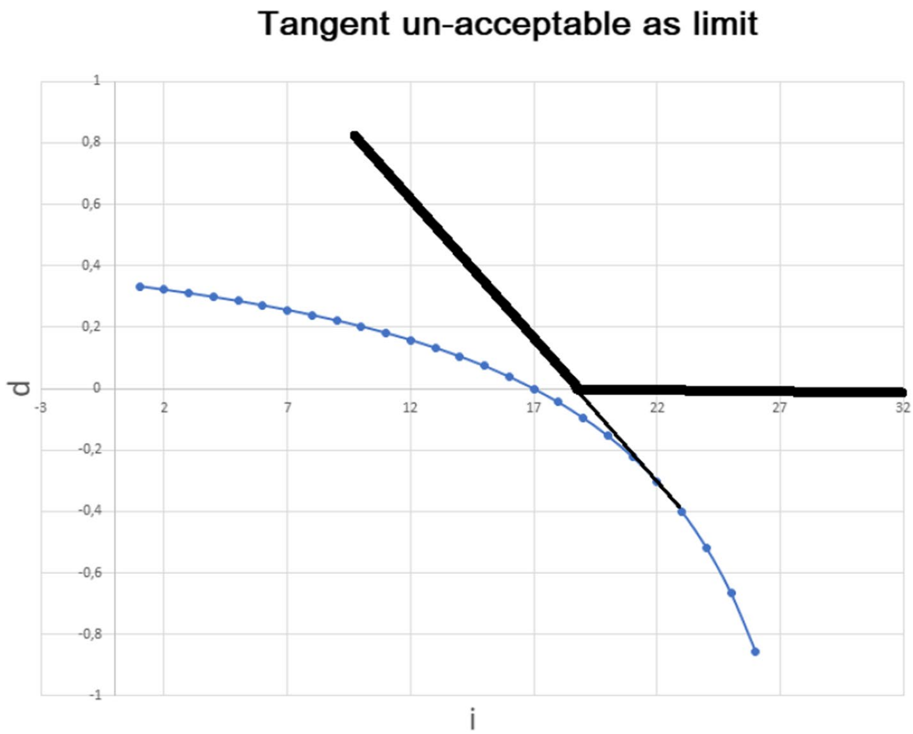

Fig. 10 Tangent linear distributions with tangent-point *d*-values lower than 0 are not distributions at the limit of collapse

($n+1$-f_0n), $i=17$ in Fig. 9, we obtain an upper value for the admissible d_{max} related to distributions at the limit of collapse:

$$d_{\max,\sup} = \frac{1 - f_0}{\alpha f_0}$$

On the other hand, the d_{max} related to the limit of collapse must surely be higher or equal to d*(1), and so

$$d_{\max,\inf} = \frac{1 - f_0}{\alpha}$$

We can thus arrive at the final conclusions if a linear distribution of damage is assumed:

1. If d_{max} is lower than $d_{max,inf}$ there is no collapse, the first strand is not broken, and so are the others.
2. If d_{max} is between $d_{max,inf}$ and $d_{max,sup}$, the limit condition is met when i_{lim} of the linear distribution (Eq. 3) is equal to expression (9).
3. If d_{max} is higher than $d_{max,sup}$ then the linear distribution is above the limit of collapse.

It should be emphasized that the condition outlined here (tangency to d^*) must also be satisfied by any other possible real distribution of damage. The method is general and can also be applied graphically.

5.3 Experimental data

To the author's knowledge there is no published available data referring to the collapse of isolated *sets* of corroded strands, and providing the amount of measured corrosion damage for each of the unit after the collapse. The studies normally deal with the load carrying capacity of *single* corroded strands and do not study the behavior of the set, with unequal corrosion.

The results of this section, however, stem necessarily from the assumptions. In particular:

1. That the stiffness of the units, if long, is not much affected by corrosion.
2. That the load carrying capacity of a single units is governed by Eq. (1).
3. That the real distribution of damage can be assumed linear, as in the case of the Polcevera viaduct.

Missing the experimental data, the next section provides an example of application for type 1 sets.

5.4 Example

A set of 32 strands initially loaded at $f_0 = 0.5$ is corroded after 20 years of service. The best possible estimate of the amount of corrosion is that the maximum loss is $d_{max} = 0.25$, while the first unit with no corrosion is $i_{lim} = 20$. Provide an engineering estimate of the remaining life of the set.

The following data are immediately computed:

$$
\begin{aligned}
d_{\max,\inf} &= 0.333 \\
d_{\max,\sup} &= 0.666
\end{aligned}
$$

At the moment, $t = t_1$, the set is safe because $d_{max} < d_{max,inf}$. To provide an estimate of the remaining life, we apply an equal increasing factor k to both d_{max} and i_{lim}. Then, Eq. (9) is rewritten as

$$
k i_{\lim} = 1 + \frac{2\alpha k d_{\max} n}{2\alpha k d_{\max} - 2 + 4f_0 + 4\sqrt{f_0^2 + \alpha k d_{\max} f_0} - f_0} \tag{10}
$$

where $f_0 = 0.5$, $n = 32$, and $\alpha = 1.5$. $i_{lim} = 20$ and $d_{max} = 0.25$. The unknown is now k and can be found numerically as $k = 1.35$.

If it is assumed that at time $t = t_0$, the damage is null, while at time $t = t_1$, the damage has some value d, two possible variations in the damage with time can be assumed.

The first assumes that the damage increases linearly with time. In this case, if as before $k = d(t_2)/d(t_1)$, and β is a suitable constant

$$d(t_2) = \beta \cdot (t_2 - t_0)$$
$$d(t_1) = \beta \cdot (t_1 - t_0)$$

which means

$$t_2 - t_1 = (k - 1) \cdot (t_1 - t_0)$$

The second assumes that the damage increases quadratically with time. In this case,

$$d(t_2) = \beta \cdot (t_2 - t_0)^2$$
$$d(t_1) = \beta \cdot (t_1 - t_0)^2$$

which means

$$t_2 - t_1 = \left(\sqrt{k} - 1\right) \cdot (t_1 - t_0)$$

In the following examples, the quadratic assumption will be used.

Therefore, according to this estimate, the collapse is expected after a time span from now equal to

$$\Delta t = t_{collapse} - t_1 = (\sqrt{k} - 1) \cdot 20\, years = 3.2\, years$$

However, this estimate has not been obtained using safety factors, which is due. Introducing a reasonable safety factor equal to 1.25 for both d_{max} and i_{lim}, we obtain "design" values:

$$d_{max,d} = 0.3125$$

$$i_{lim,d} = 25$$

and now Eq. (10) must be solved for k renamed k_d and introducing $i_{lim,d}$ and $d_{max,d}$ as i_{lim} and d_{max}, respectively. The value of k_d obtained is $k_d = 1.08$. So

$$\Delta t_d = t_{collapse,d} - t_1 = (\sqrt{k_d} - 1) \cdot 20\, years = 0.8\, years$$

which means that retrofitting or replacement is urgent.

The percentage of total area lost at the collapse condition can be evaluated as the area of the triangle having a base of $1.08(i_{lim,d}-1)$ and a depth of $1.08 d_{max,d}$ divided by $n = 32$:

$$\xi = \frac{1.08 \cdot 0.3125 \cdot 1.08 \cdot (25 - 1)}{2 \cdot 32} \approx 0.14$$

that is, 14%.

According to the uncorroded-part fallacy, this should have been 50%.

6 Load carrying capacity of binary systems under tensile force (type 2)

6.1 Worst distribution of damage

A concrete core with area A_{co} is under a compressive stress σ_0 (< 0) at time t_0. This compressive stress is the result of a preload and, after that, of the application of an external tensile axial force. At the same time, the load of a set of n strands embedded in concrete and bonded to it is f_0. This load is the sum of the needed preload applied at unbonded units, plus an additional tensile force due to the externally applied axial force, applied after bond is effective. Let m be the ratio between the Young's modulus of steel and the Young's modulus of concrete: it usually has a value of 10 or more, considering creep and long-term effects.

Corrosion attacks the strands, and after some time, some of them break. Along the recovery length L_r, at both sides of the breaking section, the tensile force lost by the broken *strands* is transferred to the concrete core, decompressing it, and to the other unbroken strands, whose load increases.

After a sufficiently high number of strands have broken, the concrete turns from being compressed to being pulled until it reaches its tensile stress σ_t, and it breaks.

When the concrete breaks, the tensile force it takes is transferred abruptly to the remaining corroded or uncorroded strands, and they behave now like the isolated set of type 1, whose mechanical behavior has been studied in the previous section.

Let b be the number of broken strands at a given time t and f be the load level of the remaining strands. Let db be the differential increment of broken strands: the force lost by them is $R_0f\mathrm{d}b$. This force is taken both by concrete and by the remaining unbroken strands (n-b), proportionally to their stiffness. Therefore, the increment of force in the concrete core is (positive and tensile)

$$dF_{co} = d\sigma_c A_{co} = R_0 fdb \cdot \frac{A_{co}}{A_{co} + m \cdot (n-b)A_{s0}}$$

where A_{s0} is the area of the unbroken strand.

The total increment of force in the remaining strands is

$$dF_s = (n-b)R_0 df = R_0 fdb \cdot \frac{(n-b)mA_{s0}}{A_{co} + (n-b)mA_{s0}}$$

Therefore, we can write the following two differential equations:

$$d\sigma = \frac{R_0 fdb}{A_{co} + m \cdot (n-b)A_{s0}}$$

$$df = fdb \cdot \frac{mA_{s0}}{A_{co} + (n-b)mA_{s0}}$$

which are solved by observing that:

- The second is easily solved by *dividing* both members by f and observing that the integral of df/f is ln(f)
- The first is then solved by replacing the f found from the second.
- The two integration constants are found by imposing that for $b=0$, $f=f_0$ and $\sigma=\sigma_0$.

Therefore, the following functions $f(b)$ and $\sigma(b)$ are found:

$$f = f_0 \cdot \frac{nmA_{s0} + A_{co}}{A_{co} + (n-b)mA_{s0}} \tag{10a}$$

$$\sigma - \sigma_0 = \frac{R_0 f_0 \cdot (nmA_{s0} + A_{co})}{mA_{s0} \cdot [(n-b)mA_{s0} + A_{co}]} - \frac{R_0 f_0}{mA_{s0}} \tag{10b}$$

The first of these two relationships, (10a), is the equivalent of Eq. (4) for type 1 elements: it states how the load increases with the increase in the number of broken strands. It holds true up to the point when the concrete reaches the tensile ultimate stress σ_t, that is, when the number of broken strands reaches the number

$$b_{break,c} = \frac{(\sigma_t - \sigma_0) \cdot (nmA_{s0} + A_c)}{R_0 f_0 + (\sigma_t - \sigma_0) \cdot mA_{s0}} \tag{11}$$

When the concrete breaks, there can be no distribution to concrete anymore, and from that point onward, the remaining (n-$b_{break,c}$) strands behave like an isolated set (type 1). The load level f of the strands after the break changes abruptly from

$$f_{break,-} = f_0 \cdot \frac{nmA_{s0} + A_{co}}{(n - b_{break,c})mA_{s0} + A_{co}}$$

to

$$f_{break,+} = f_{break,-} + \frac{\sigma_t A_{co}}{\left(n - b_{break,c}\right) \cdot R_0}$$

From the cracking of concrete onward, the load f of each remaining strand increases with the number of broken strands following the function

$$f = \left[f_0 \frac{nmA_{s0} + A_{co}}{\left(n - b_{break,c}\right)mA_{s0} + A_{co}} + \frac{\sigma_t A_{co}}{\left(n - b_{break,c}\right) \cdot R_0}\right] \cdot \frac{\left(n - b_{break,c}\right)}{(n-b)} \text{ with } b > b_{break,c} \tag{12}$$

Following the path of type 1 systems, we can now evaluate the worst possible distribution of damage between the strands, which implies that when the first strand breaks, all the other strands also break, one after another. To this aim, we set for the first stage (uncracked concrete, Eq. 10a)

$$f = f_0 \cdot \frac{nmA_{s0} + A_{co}}{A_{co} + (n-b)mA_{s0}} = 1 - \alpha d_{b+1} = r_{b+1}$$

In addition, replacing $b+1$ by i, we obtain the worst distribution of damage, which is valid for $i=1$ to $i=b_{break,c}+1$:

$$d_i^* = \frac{1}{\alpha} \cdot \left[1 - f_0 \frac{nmA_{s0} + A_{co}}{(n-i+1)mA_{s0} + A_{co}}\right] i = 1 \text{ to } b_{break,c} + 1 \tag{13}$$

After the breaking of concrete, using Eq. 12, we obtain the worst distribution of damage for the remaining strands:

$$f = \left[f_0 \frac{nmA_{s0} + A_{co}}{\left(n - b_{break,c}\right)mA_{s0} + A_{co}} + \frac{\sigma_t A_{co}}{\left(n - b_{break,c}\right) \cdot R_0}\right] \cdot \frac{\left(n - b_{break,c}\right)}{(n-b)} = 1 - \alpha d_{b+1} = r$$

Therefore, the worst damage distribution after the breaking of concrete is

$$d_i^* = \frac{1}{\alpha}\left[1 - \left(f_0 \frac{nmA_{s0} + A_{co}}{(n - b_{break,c})mA_{s0} + A_{co}} + \frac{\sigma_t A_{co}}{(n - b_{break,c}) \cdot R_0}\right) \cdot \frac{(n - b_{break,c})}{(n - i + 1)}\right]$$

with $i \geq b_{break,c} + 1$.

The worst damage curve has a discontinuity for $i = b_{break,c} + 1$, which is related to the sudden cracking of concrete (see. Fig 11).

In Fig. 11, the worst distribution of damage for an identical set of strands identically loaded at $t = t_0$ for Type 1 and Type 2 structural elements is plotted. The protective effect of the concrete (Type 2) is evident, which implies a distribution of worst damage with much higher values than that without concrete (Type 1).

However, this protection effect has a drawback: it can end abruptly, resulting in a sudden extra load to the remaining strands, a condition that is to be considered for most parts of the distributions, the immediate onset of the collapse.

6.2 Failure condition for real distributions of damage

As can be clearly seen looking at Fig. 12, assuming a linear real damage distribution, as in the previous section, the limit condition is met *when the linear curve passes through point* **K**, whose coordinates are

$$\mathbf{K} = \left\{ b_{break,c} + 1, \frac{1}{\alpha}\left[1 - f_0 \frac{nmA_{s0} + A_{co}}{(n - b_{break,c})mA_{s0} + A_{co}}\right] \right\}$$

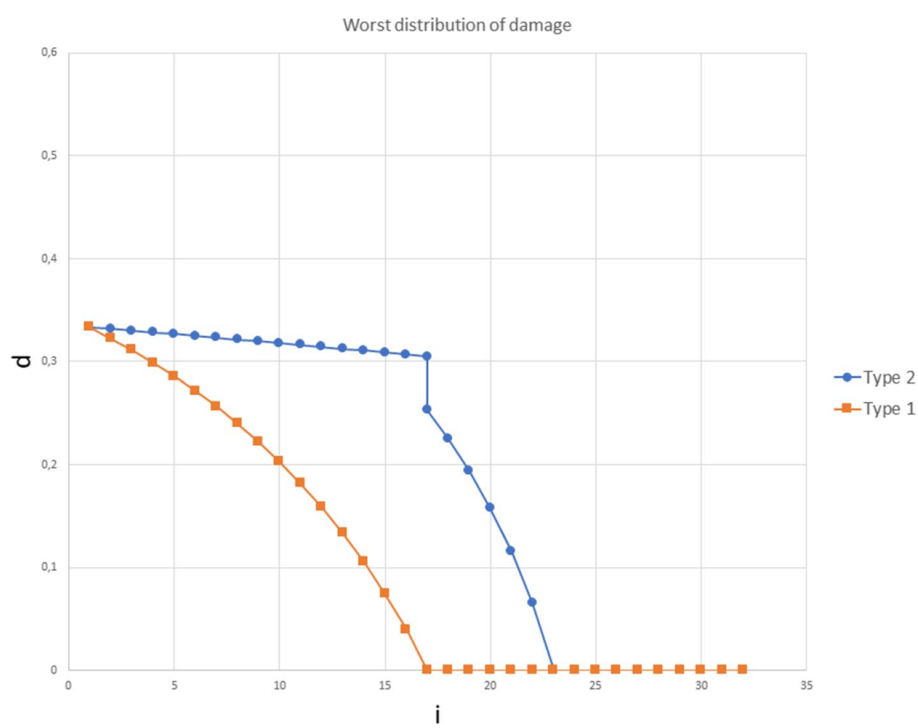

Fig. 11 Comparison between the type 1 and type 2 worst distributions of damage d*. The following data are used for this plotting: $n = 32$. $\alpha = 1.5$. $f_0 = 0.5$. $\sigma_0 = -6$ MPa. $\sigma_t = 1.24$ MPa. $A_{s0} = 93$ mm^2. $R_0 = 158.1$ kN. $A_{co} = 160{,}000$ mm.^{2}. $m = 10$

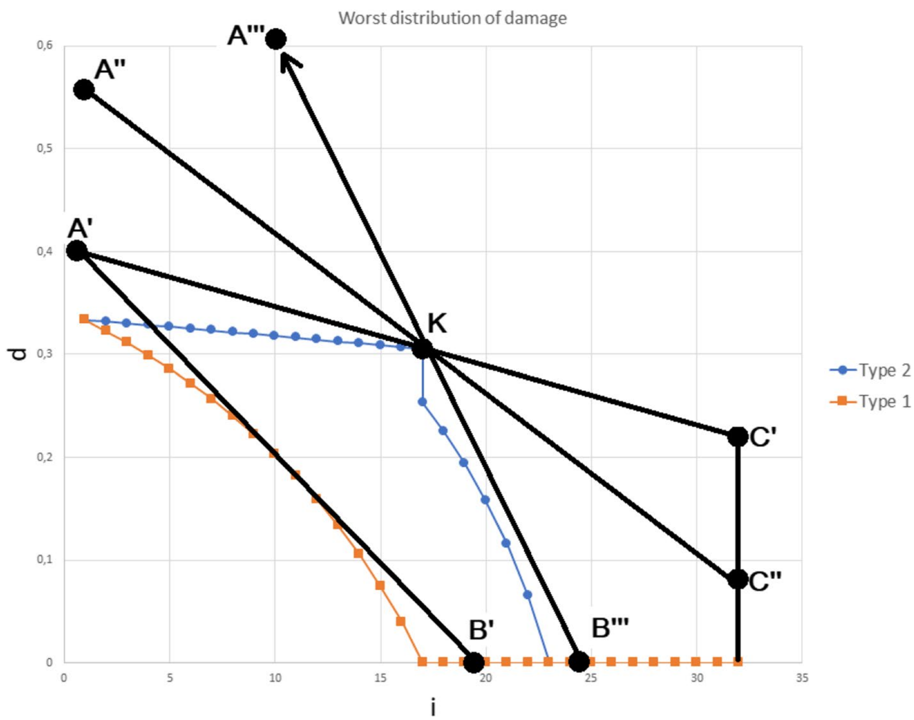

Fig. 12 Distribution of damage at collapse for Type 2 systems. Considering, for example, $d_{max}=0.4$ (point A'), it is apparent that the damage at collapse needed for systems of type 2 (A'-C') is much higher than that of type 1 (A'-B')

The linear curves having a given d_{max}, for $i=1$, must have i_{lim} equal to the following expression to also pass through point **K**:

$$i_{\lim} = 1 + \frac{d_{\max} \cdot b_{break,c}}{\left\{d_{\max} - \frac{1}{\alpha}\left[1 - f_0 \frac{nmA_{s0}+A_{co}}{(n-b_{break,c})mA_{s0}+A_{co}}\right]\right\}} \tag{14}$$

That is, a linear distribution with d_{max} is at the failure limit when its i_{lim} is equal to expression (14).

Now, if at a time $t=t_1$ the damage distribution is evaluated having a given d_{max} and i_{lim}, in safe conditions, assuming a linear variation with time of the damage, a factor k can be applied to both d_{max} and i_{lim}. Replacing d_{max} by kd_{max} and i_{lim} by ki_{lim} in expression (14), a second-order equation where k is unknown is found. In particular, introducing the position

$$U = \frac{1}{\alpha}\left[1 - f_0 \frac{nmA_{s0} + A_{co}}{(n - b_{break,c})mA_{s0} + A_{co}}\right] \tag{15}$$

The following expression for k is found:

$$k = \frac{i_{\lim}U + d_{\max}b_{break,c} + d_{\max} + \sqrt{\left(i_{\lim}U + d_{\max}b_{break,c} + d_{\max}\right)^2 - 4d_{\max}i_{\lim}U}}{2d_{\max}i_{\lim}} \tag{16}$$

By using expression (16), once the values of d_{max} and i_{lim} of the linear distribution of damage have been estimated, an engineering measure of the remaining life can be obtained. As in the example of the previous section, it is convenient to apply a proper safety factor, here proposed equal to 1.25, to both d_{max} and i_{lim}.

6.3 Experimental results

The method outlined here for type 2 systems has been used to explain the collapse of the southeast stay of balanced system 9 of the Polcevera Viaduct (Fig. 13), with good agreement with the available data.

This is a rare case where most part of the needed data have been measured. The Polcevera viaduct collapse can then be considered as a real-world experiment of collapse of type 2 corroded systems.

The following data should be used:

$$
\begin{aligned}
d_{max} &= 0.859 & (*)\\
i_{lim} &= 478 & (*)\\
\alpha &= 1.3 & (*)\\
n &= 464 & (*)\\
f_0 &= 0.4 & \\
\sigma_0 &= -6.7\text{MPa} & \\
\sigma_t &= 4\text{MPa} & (*)\\
A_{s0} &= 93\text{mm}^2 & (*)\\
A_{co} &= 1220 * 980\text{mm}^2 - 464 * 93\text{mm}^2 = 1152448\text{mm}^2 & (*)\\
m &= 10 & \\
R_0 &= 1800\text{MPa} * 93\text{mm}^2 = 167.4\text{kN} & (*)
\end{aligned}
$$

- The data marked by an asterisk derive from *measures* taken during the forensic analysis.
- The values of f_0 and σ_0 are not known with exactitude, and the design values have been used.
- The value of m is a reasonable guess; however, the result is slightly dependent on this value.
- The values d_{max} and i_{lim} stem from the measures of the broken wires read after the collapse; as already explained, no safety factor is applied (squared line of Fig. 13, which is the dotted line of Fig. 5).

With these values:

$$b_{break,c} \approx 220;\ U = 0.415;\ k = 0.948.$$

and

$$
\begin{aligned}
t_{collapse} - t_{2018} &= \left(\sqrt{0.946} - 1\right) \cdot (t_{2018} - t_{1967})\\
t_{collapse} &= 2018 - 0.0273 \cdot 51 \approx 2016.6
\end{aligned}
$$

Considering the uncertainness related to the values of σ_0 and f_0, i.e., the initial values of load in the strands and in the concrete, applied at construction, the other uncertain data and the vicinity of the value of k obtained to 1.0 (difference of 0.053, that is, only 5.3%),

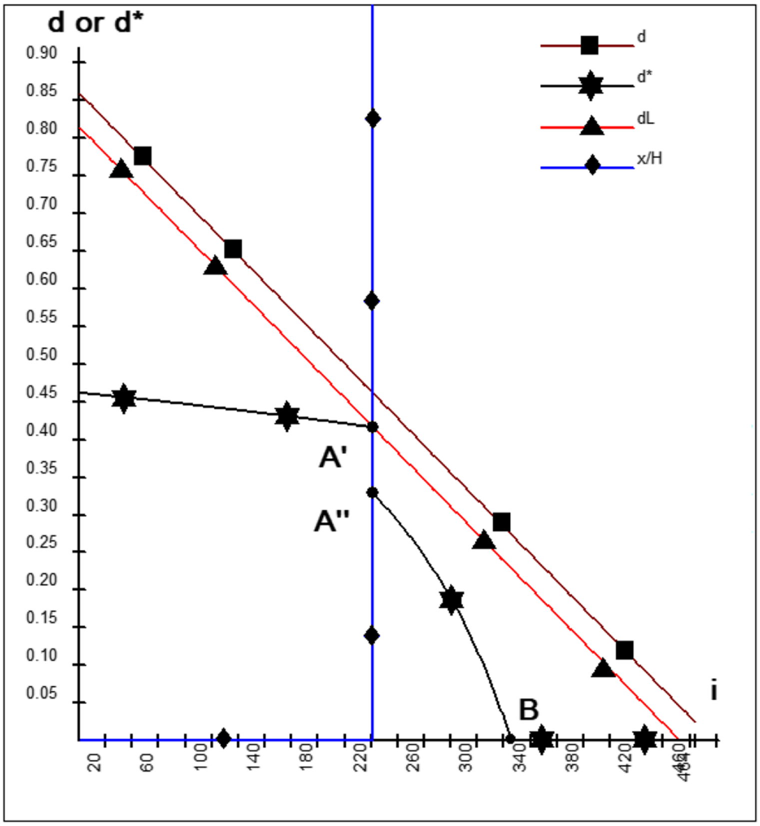

Fig. 13 The plotting of $d^*(i)$, $d(i)$ and $d_{lim}(i)$ for the Polcevera balanced system 9, southeast stay, at the breaking section. The diamond curve is the relative crack depth, i.e., 0 or 1. The concrete breaks at point **A′** = **K**. At point **A″**, only the steel is reacting

this, in the author's opinion, is to be considered an engineering exact explanation of the collapse that occurred on August 14, 2018.

Indeed, the method here presented has been obtained by the author after the in-depth study of the collapse.

So, the collapse of the Polcevera viaduct is to be considered a tragic experimental test. The study of this tragic test led to the method here proposed:

- the real distribution of damage in the strands was measured wire by wire and it was linear (square curve Fig. 12);
- it reached the point **K** = **A′** of the worst distribution of damage (star curve Fig. 12) with very good agreement.
- The concrete suddenly broke, and the progressive type 1 rupture of all the remaining strands followed immediately. After the point **A′** = **K** the real distribution of damage (square) is much higher than the worst one (star).
- The breaking of the stay caused the collapse of the whole "balanced system number 9", as the balance of the forces was suddenly lost.

The collapse caused the death of 43 people.

7 Load carrying capacity of binary systems under a bending moment and an axial force(type 3)

7.1 Worst distribution of damage

A preloaded concrete section, after the application of an external positive bending moment M, is such that the n steel units of area A_{s0} (strands or possibly single wires) are all loaded at f_0, i.e., after the application of moment M; the compressive (negative) stress at the bottom of the concrete section is σ_{bot0}. The data needed are f_0 and $\sigma_{bot,0}$ at time $t=t_0$. The method and the equations are also valid if the applied external loads are a bending moment M and an axial force N.

It will be assumed that the bending moment M and the axial force N, if applied, are kept constant from time $t=t_0$ when there is no corrosion to time $t=t_1$, when a distribution of damage due to corrosion is detected. The bending M could be safely assumed to be the sum of permanent and variable loads, considering a reasonable amount for the latter (e.g., rare or frequent values).

With reference to Fig. 14, the following new symbols are introduced:

- e_c the distance of the concrete section center to the bottom extremity (no steel considered, 1843.6 mm for the section in Fig. 14).
- J_c the second area moment of the concrete section to its center axis (0.9862 m^4 for the section in Fig. 14).
- A_{c0} concrete section area (1.345 m^2 for the section in Fig. 14).
- c the distance of the steel center to the bottom side (65 mm in Fig. 14).
- H the depth of the Sect. (2500 mm in Fig. 14).
- B_1 the width of the section at bottom (650 mm in Fig. 14, see also Fig. 15).
- B_2 the width of the web, or in general, a second width (150 mm in Fig. 14).
- H_1 the depth of the section where $B=B_1$ is constant (200 mm in Fig. 14).
- H_2 the depth of the transition zone from width B_1 to width B_2 (150 mm in Fig. 14).

Considering also the steel, the following expressions can be found for the center position of the concrete+steel section, e_0, its area A_0, and its second area moment to the binary section center-axis J_0:

$$e_0 = \frac{A_c e_c + mnA_{s0}c}{A_c + mnA_{s0}}$$

$$A_0 = A_c + mnA_{s0} \tag{17}$$

$$J_0 = J_c + A_c \cdot (e_0 - e_c)^2 + mnA_{s0} \cdot (e_0 - c)^2$$

where the homogenization factor m has been used.

During the corrosion process, a number b of units are broken, and the section may be cracked. It will be assumed that the position of the crack is x, i.e., the distance of the limit cracked-uncracked to the bottom side (Fig. 15).

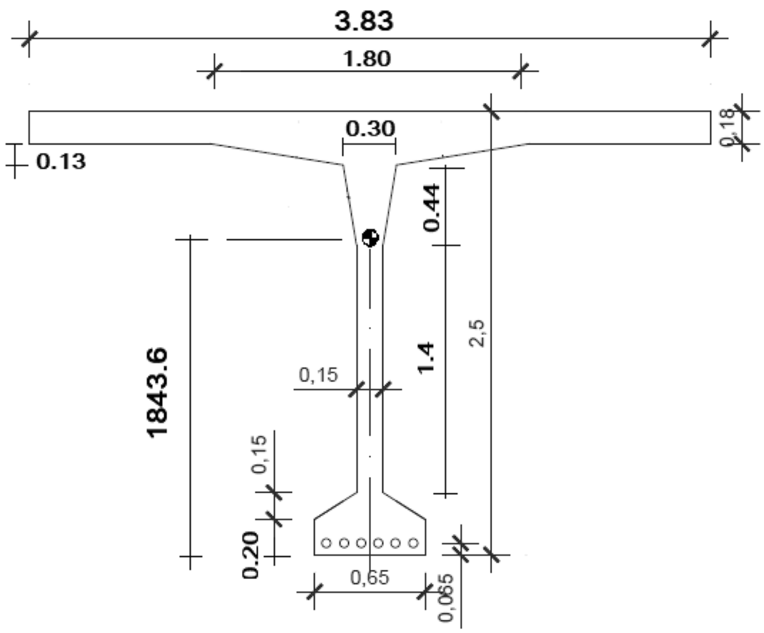

Fig. 14 A bridge preloaded concrete section. Sizes in meters

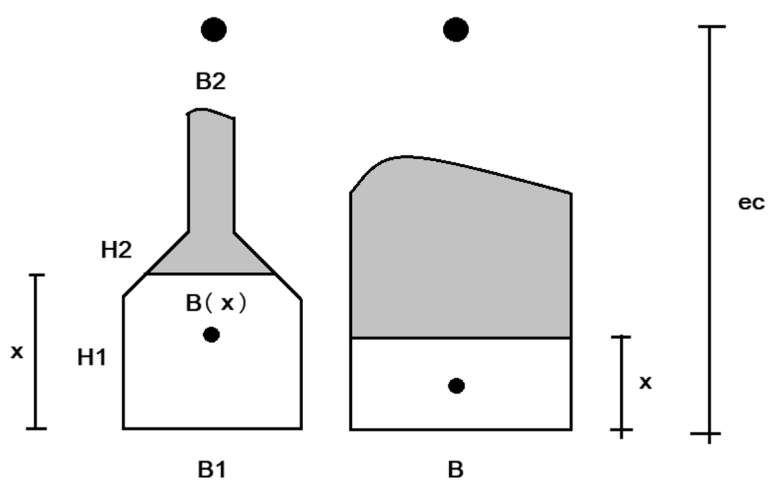

Fig. 15 The detail of a cracked section. *x* is the depth of the crack

The expressions of the center position, area and second area moment of the cracked section with some units broken must then be modified as follows:

$$e(x,b) = \frac{A_c e_c - S_{loss}(x) + mnA_{s0}c - bmA_{s0}c}{A_c + mnA_{s0} - bmA_{s0}} \approx e(x) = e_x = \frac{A_c e_c - S_{loss}(x) + mnA_{s0}c}{A_c + mnA_{s0}}$$

$$A = A_c - A_{loss}(x) + mnA_{s0} - mA_{s0}b$$

$$J = J_c + A_c \cdot (e_x - e_c)^2 - J_{loss}(x) + mnA_{s0} \cdot (e_x - c)^2 - mA_{s0} \cdot (e_x - c)^2 b$$

Defining the helper quantities

$$A_x = A_c - A_{loss}(x) + mnA_{s0}$$

$$J_x = J_c + A_c \cdot (e_x - e_c)^2 - J_{loss}(x) + mnA_{s0} \cdot (e_x - c)^2$$

We obtain

$$A = A_x - mA_{so}b$$

$$J = J_x - mA_{s0} \cdot (e_x - c)^2 b$$

A_{loss}, S_{loss} and J_{loss} are the losses of area, first area moment and second area moment of the concrete section due to the crack, respectively. They are all functions of x.

In the expression of e_x, the dependence on b has been neglected, assuming $e(x, b) \approx e(x)$.

For the easiest case of a rectangular section, the losses are

$$A_{loss}(x) = B_1 x$$
$$S_{loss}(x) = B_1 x \cdot \frac{x}{2}$$
$$J_{loss}(x) = \frac{1}{12} B_1 x^3 + B_1 x \cdot \left(e_x - \frac{x}{2}\right)^2$$

The more complex case of a bulb at the bottom is omitted, but it is straightforward.

If the differential of a unit breaks, db, when it is at a load level f, at a generic cracked or not cracked configuration, the force and moment lost are

$$dF = fdbR_0$$
$$dM = fdbR_0 \cdot (e_x - c)$$

The variation in f in the steel and compressive/tensile stress at the cracked-uncracked interface due to this force and moment can be found by

$$df = \frac{fdbR_0}{A} \cdot \frac{mA_{s0}}{R_0} + \frac{fdbR_0 \cdot (e_x - c) \cdot (e_x - c)}{J} \cdot \frac{mA_{s0}}{R_0} = fdb \frac{mA_{s0}}{A_x - bmA_{s0}} + fdb \frac{mA_{s0} \cdot (e_x - c)^2}{J_x - mA_{s0} \cdot (e_x - c)^2 b}$$

$$d\sigma = \frac{fdbR_0}{A} + \frac{fdbR_0 \cdot (e_x - c) \cdot (e_x - x)}{J} = fdbR_0 \frac{1}{A_x - mA_{s0}b} + fdbR_0 \frac{(e_x - c) \cdot (e_x - x)}{J_x - mA_{s0} \cdot (e_x - c)^2 b}$$

This is a system of two coupled partial differential equations that can be solved as follows. The first provides $f = f(b,x)$. It has the solution

$$\ln f = -\ln(A_x - mA_{s0}b) - \ln\left(J_x - mA_{s0} \cdot (e_x - c)^2 b\right) + C_1$$

where C_1 is a constant that can be found by imposing that for $b = 0$ and $x = 0$, $f = f_0$. Therefore, the following expression for f is found (if f is found to be greater than 1, collapse has been reached, with no damage in the remaining units):

$$f = \frac{f_0 A_0 J_0}{(A_x - mA_{s0}b) \cdot \left(J_x - mA_{s0} \cdot (e_x - c)^2 b\right)} \leq 1 \tag{18}$$

Replacing this f in the differential equation of $d\sigma$, we obtain the following differential equation:

$$d\sigma = R_0 f_0 A_0 J_0 \cdot \left\{ \frac{db}{(A_x - mA_{s0}b)^2 \left[J_x - mA_{s0} \cdot (e_x - c)^2 b\right]} + \frac{(e_x - x) \cdot (e_x - c) db}{(A_x - mA_{s0}b) \left[J_x - mA_{s0} \cdot (e_x - c)^2 b\right]^2} \right\}$$

This can be solved as

$$\sigma = \frac{R_0 f_0 A_0 J_0}{mA_{s0} \left[J_x - A_x \cdot (e_x - c)^2\right]^2} \cdot \left\{ \left[(e_x - x) \cdot (e_x - c) - (e_x - c)^2\right] \cdot \ln \left[\frac{J_x - mA_{s0}(e_x - c)^2 b}{A_x - mA_{s0}b}\right] + \frac{J_x - A_x \cdot (e_x - c)^2}{A_x - mA_{s0}b} - \frac{\left[J_x - A_x \cdot (e_x - c)^2\right] \cdot (e_x - x) \cdot (e_x - c)}{J_x - mA_{s0} \cdot (e_x - c)^2 b} \right\} + C_2 \tag{19}$$

The constant C_2 can be found by observing that the function of x and b $\sigma(x, b)$ should be equal to σ_{bot0} for $x = 0$ and $b = 0$: $\sigma(0, 0) = \sigma_{bot0}$. *So*

$$C_2 = \sigma_{bot0} - \frac{R_0 f_0 A_0 J_0}{mA_{s0} \left[J_0 - A_0 \cdot (e_0 - c)^2\right]^2} \cdot \left\{ \left[e_0 \cdot (e_0 - c) - (e_0 - c)^2\right] \cdot \ln \left[\frac{J_0}{A_0}\right] + \frac{J_0 - A_0 \cdot (e_0 - c)^2}{A_0} - \frac{\left[J_0 - A_0 \cdot (e_0 - c)^2\right] \cdot e_0 (e_0 - c)}{J_0} \right\}$$

The number of broken units related to the first cracking of concrete can be found by the final expression $\sigma = \sigma(x, b)$ by imposing that the stress at the bottom ($x = 0$) is the limit tensile stress σ_t (which can also be 0):

$$\sigma_t = \sigma(0, b)$$

This equation can be solved numerically. Let $b = b_{\mathrm{break}}$ its solution. When the number of broken units reaches this number, the concrete bottom side is at the limit of cracking.

From that point onward, given a value of $b = b_{cur} > b_{break}$, the unknown is x. For each b_{cur}, the related x_{cur} is found by numerically solving the equation

$$\sigma_t = \sigma(x, b_{cur})$$

Now, since we have x_{cur} related to b_{cur}, we can replace x by x_{cur} and b by b_{cur} in expression (18), where A_x and J_x are known functions of x, to find the value f_{cur} of the load of the remaining unbroken strands. Setting up a loop with b ranging from = 0 to n-1, we have the algorithm of Table 2 to find the worst distribution of damage $d^*(i)$.

7.2 Experimental data

To the author's knowledge there is no available complete data referring to tests of prestressed beams loaded to collapse, having the steel part corroded. In particular, what would be necessary in order to validate the method, and it is not available, is the measure of the damage of all the steel units after the collapse. To the author's knowledge this information is not available in the published tests. Ideally, these tests should be carried on using real beams, having the steel part corroded. The author believes these test are urgently necessary, considered the high number of existing bridges with resisting steel units corroded.

Table 2 Algorithm to find the worst distribution of damage for Type 3

b	*f*	*d**
$0 \le b_{cur} \le b_{break}$	Use (18) with $x=0$ to find f_{cur}	$d^*_{b+1} = d^*_i = \frac{1-f_{cur}}{\alpha}$
$b_{cur} > b_{break}$	Use (19) with $\sigma=\sigma_t$ and $b=b_{cur}$ to find $x=x_{cur}$ Use (18) with $\sigma=\sigma_t$, $x=x_{cur}$ and $b=b_{cur}$ to find f_{cur}	$d^*_{b+1} = d^*_i = \frac{1-f_{cur}}{\alpha}$

Table 3 Data for example 1

n	A_{s0} (mm^2)	*m*	α	f_0	R_0 (N)	σ_{bot0} (MPa)	d_{max}	i_{lim}	t_1-t_0 years
252	28.27	15	1.3	0.718	48,059	-3.78	0.2	200	20

The results obtained in this section are a direct formal consequence of the main assumptions, which in part are the same used for systems of type 1 and 2, and that all seem reasonably correct i.e.:

- That the overall global stiffness of long steel units is not much affected by corrosion.
- That the load carrying capacity of single units can be expressed by (1).
- That the steel units are sufficiently near one another to neglect the differences in their position.
- That the real distribution of damage between the units is linear, like that found in the Polcevera stay. However, the method main result is the worst distribution of damage for type 3 systems, which is a property of the original structure. Every real distribution of damage can be checked against it.

In the author opinion the method presented here can be used for engineering estimates of the safety of existing structures, provided that safe-side estimates of the real distribution of damage are used.

Lacking experimental data to compare to, in the next section two ideal examples will be provided.

7.3 Example 1

Assume the use of the concrete section of Fig. 14, which is a real concrete bridge section. The values of Table 3 are used: the linear damage distribution data are fictious.

These are coherent with a total external applied bending moment of 14,125 kNm. The part due to (maximum) live loads is $M_q = 7{,}828$ kNm.

The results are plotted in Fig. 16. Considering the worst damage distribution (star curve, Fig. 16), they are the following:

- The number of broken units at first cracking of concrete is 73 (point A).
- The number of broken units at complete collapse is 99 (point B).
- Initially, the worst damage curve d* (star curve of Fig. 16) decreases with the increase in the broken units, with a curve having a relatively limited slope (up to point A). After the concrete breaks, the section softens, and the slope of the worst damage

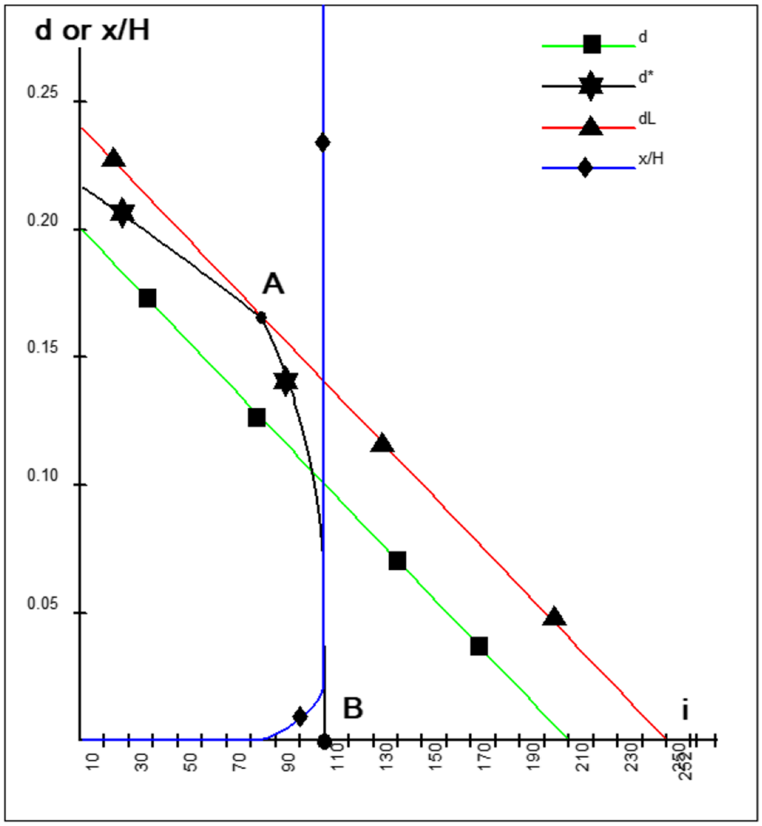

Fig. 16 Example 1. The worst damage distribution d* (star curve), the assumed damage distribution (square curve), the collapse damage distribution (triangle) and the crack opening *x/H* (diamond)

curve diminishes sharply and continuously (A-B). The depth of the crack x/H is plotted in the diamond curve. At the point of collapse, B, $x=49.14$ mm.

- The total corroded area loss of the worst damage distribution is only 7%.
- The collapse is reached when the load f of the unbroken wires reaches 1. At that point, all the remaining strands break, at 0 damage in the worst damage distribution (axis i).

Assuming a (fictious) linear damage distribution, $d_{max}=0.2$ $i_{lim}=200$ (no safety factor applied), and the factor k is 1.2. with a quadratic dependence on time ($t_{collapse}$-t_1)$=1.9$ years. The collapse is reached when the limit damage distribution passes through point A. The limit-damage-distribution (triangle curve) corroded-area loss, is (0.2x1.2x200x1.2)/(2x254)= 0.113, that is 11.3%. The collapse, which is immediate and fragile, is reached when the corroded area loss is slightly over 10%.

7.4 Example 2

This example is similar to example 1, but the live loads have been halved, so the bending moment applied is 10,211 kNm. The values of f_0 and σ_{bot0} change accordingly (see Table 4).

The following results have changed (see also Fig. 17):

Table 4 Data for example 2

n	A_{s0} (mm²)	m	α	f_0	R_0 (N)	σ_{bot0} (MPa)	d_{max}	i_{lim}	t_1-t_0 years
252	28.27	15	1.3	0.674	48,059	-5.94	0.2	200	20

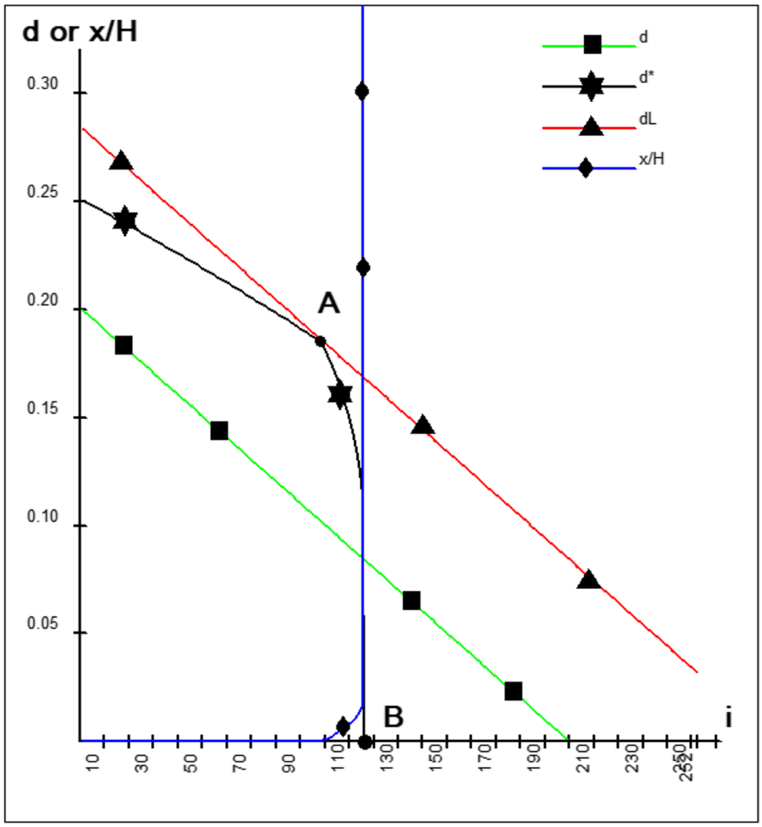

Fig. 17 Example 2. The worst damage distribution d* (star curve), the assumed damage distribution (square curve), the collapse damage distribution (triangle) and the crack opening *x/H* (diamond)

- The number of broken units at first cracking of concrete is now 97 (point A).
- The number of broken units at complete collapse is now 115 (point B).
- The opening of the crack at the point of collapse, B, is now $x = 39.2$ mm.
- The total corroded area loss of the worst damage distribution (star curve) is now 9.7%.
- The k factor is now 1.42.
- The time to collapse with quadratic time variation is now $(t_{collapse}\text{-}t_1) = 3.8$ years.

The results are plotted in Fig. 17.

8 Conclusions

The collapse of sets of corroded strands or wires depends on the distribution of damage between them and not on the total corroded area loss.

In particular, in all three cases studied, an isolated set of strands (type 1), strands in a concrete core under a tensile force (type 2), and strands in a concrete core under a bending moment and axial force (type 3), collapse is immediately triggered when the distribution of damage between the strands reaches the condition that for each strand i, the damage of strand d_i is greater than or equal to the damage of the worst distribution of damage, d^*_i.

It is therefore totally groundless to assume that the load carrying capacity of the corroded strands is related to their overall corroded area loss. The use of such a fallacy could result in very dangerous safety estimates.

The worst distribution of damage has been written in closed form for type 1 and 2 systems, while for type 3, only part of it is available in closed form, i.e., up to the first cracking of concrete. From there onward, the curve must be found numerically. By plotting reasonable and safe real damage distribution estimates, against the worst damage distribution, an engineering evaluation of the safety is possible.

This is particularly useful in the evaluation of existing structures, where corroded steel units are in working condition.

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Authors' contributions
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Availability of data and materials
The data used for the example in section 6.3 are available in the documents Rugarli 2020a and Rugarli 2020b. These are part of the public documents of the ongoing trial for the Morandi Bridge Collapse: ***Procedimento Penale N. 10468-18 R. G. N. R., N.7998-18 R.G. N. R., Tribunale di Genova***.

Declarations

Competing interests
The author has no competing interest.

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