# Quantitative analysis of the importance and correlation of urban bridges and roads in the study of road network vulnerability 



*Correspondence:
hongweih@tongji.edu.cn
${ }^{1}$ Department of Bridge Engineering, Tongji University, Shanghai 200092, China
${ }^{2}$ State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China


#### Abstract

The city development is closely related to the performance of the transportation network system. Bridges and roads are important parts of the transportation system, and are also inseparable components of the transportation network. However, the effect of the correlation between bridges and roads on the network system has not been studies thoroughly in the literature. Therefore, it is necessary to analyse the vulnerability of the road network when both bridges and roads are involved. In this paper, the urban road network is modeled into the form of network connection and node, based on the analysis of the related research results of road network vulnerability in the literature. Taking the urban roads at all levels as the connection and the transportation hubs (including bridges) as the nodes, the paper puts forward the corresponding measurement indexes and calculation methods, and establishes the importance and correlation analysis model of roads and bridges in the urban road network. At last, the model is applied to the road network which is $5 \times 3 \mathrm{~km}^{2}$ besides Yangpu Bridge of Shanghai for verification, the importance and correlation of specific roads and bridges in the analyzed urban road network are calculated, which provides a certain basis for dealing with various emergencies leading to the decline of urban road network vulnerability. In this paper, the importance analysis of urban road network is extended to the bridge correlation analysis, so that the proposed model of the vulnerability assessment of the urban road network system is more suitable for the increasingly demand of road and bridge construction in China, and provides a certain basis for dealing with the decline of road network vulnerability caused by various emergencies.


Keywords: Urban road network, Transportation system, Vulnerability analysis, Importance and correlation between bridge and road

## 1 Introduction

Road traffic network vulnerability usually refers to the sensitivity of road network capacity reduction caused by emergencies, which is manifested as the cascading failure of related road sections caused by the loss of capacity of some road sections, resulting in large-scale traffic network congestion (Zhang et al. 2013). With the enrichment of

[^0]modern means of transportation and modes of transportation, the vulnerability analysis of road network has become an important work of urban road planning.

Scholars have done a lot of research on road network vulnerability. Berdica (2002) defines vulnerability by considering the probability and consequence of events that affect the road transportation system, that is, vulnerability in the road transportation system is the susceptibility of events, which can greatly reduce the maintainability of the road network. Jenelius (2008) pointed out that the purpose of road network vulnerability analysis is to assess the possibility of emergencies and their economic and social consequences, and shift the focus of work from the link level to the regional level. Jenelius et al. (2011) proposed a multi-itinerary scheduling model based on a single itinerary scheduling model, and studied the impact of scheduling flexibility and interdependence between different segments in the daily itinerary chain on delay cost and time value. Taylor and Susilawati (2012) proposed a node based vulnerability analysis and a location-based vulnerability analysis, which expanded the vulnerability analysis method of road network to a new scope. Jenelius and Mattsson (2015) believed that road network vulnerability analysis can be defined as studying the potential degradation of road transport system and its impact on society, and modeling road infrastructure as a network with links (road sections) and nodes (intersections), which provides a theoretical basis for subsequent modeling analysis of road network. Liu and Rong (2018) realized that the road network vulnerability analysis not only includes the vulnerability of its own structure and network physical vulnerability, but also should consider that some travel users give up travel due to the interruption of road sections.

As the main part of vulnerability analysis, importance analysis is relatively sufficient. Menelaus (2010) proposed a method to measure the importance of road connections, which takes the user's rights into account. Jenelius et al. (2005) pointed out that it was very difficult to estimate the possibility of extreme events such as natural disasters and terrorist attacks, and thus used the concept of exposure again, assuming that a dangerous event was a link or a group of links, collectively referred to as an element. When it is completely destroyed or closed, users on these links will be forced to take other unfavorable routes, and a calculation idea is proposed to calculate the exposure degree of specific events for a single demand node, a group of demand nodes or the whole network. The mode of road network destruction is not only the destruction or closure of a single link, but also the interruption of the road network caused by the destruction of regional coverage. In view of this, Jenelius and Mattsson (2012) proposed a comprehensive method to study the vulnerability of road network under the destruction of regional coverage. By analyzing the elastic demand network model and fixed demand network model, Qiang and Nagurney (2008) proposed a new and unified network performance measurement method, which can easily evaluate the importance of network nodes and links. In contrast, the network performance measurement method proposed by Latora and Marchiori (2004) (also known as L-M method) and the measurement method proposed by Zhu et al. (2006) have some limitations, which are convenient methods in specific situations. The theoretical basis of importance analysis is the shortest path algorithm. Han (2017) focused on the Dijkstra algorithm for solving the shortest path of the urban road network and the Yen algorithm for solving the K shortest path. He proposed to use the adjacency list data structure to store the urban road network map, reducing
the spatial complexity of the Dijkstra algorithm. Gan et al. (2017) improved the traditional Dijkstra algorithm, starting from a specific node, and then calculated the shortest path between each node, so as to simplify the algorithm. Li et al. (2004) used the sum of the reciprocal distances (shortest paths) between all disconnected node pairs formed after node (set) deletion to reflect the degree of network connectivity damage caused by node deletion, that is, the importance of the deleted node (set).
In fact, vulnerability analysis consists not only importance analysis, but also correlation analysis. At present, there is little work on correlation analysis at home and abroad, and its research method is not very clear. Therefore, this paper considers from the perspective of the establishment and application of measurement indicators. Potts (1994) introduced the concept of node degree, that is, the number of all links connected to a node. Freeman (1978) proposed the proximity centrality, which refers to the reciprocal of the sum of the shortest distances from a node to all other nodes in the road node. Bonacich (1972) studied the intermediary centrality, which means the number of nodes passing through the shortest path between two nodes in the road network. Each measurement index has independent significance. Xiao (2019) applied the data processing method by studying and using the grey correlation analysis method. The theory of congestion propagation and the idea of dynamic traffic allocation was applied to evaluate the degree of traffic damage in emergency situations, and proposed vulnerability evaluation indicators based on traffic conditions and road network topology (Tong et al. 2017; Sun 2019).
Previous studies on the vulnerability of road networks have mostly focused on the importance aspect, that is, the impact of the efficiency of each link in the road network on the entire road network. This article first expands the concept of road network vulnerability from importance to correlation. In this article, importance study means the path and region importance degree in the road network; As for the correlation study, it contains two parts, the one is the corrrelation between bridge and the whole road network, and the other is the corrrelation between bridge and other rode path. On this basis, this paper establishes the mathematical model of importance and correlation and quantify the importance and correlation by setting indicators, which can intuitively represent the importance of links in the road network and the impact of other links on key nodes such as bridges. This can provide reference for future road network planning, operation, and maintenance.

## 2 Importance and correlation

Within the scope of this paper, importance mainly refers to the impact of different urban roads on road network vulnerability, while correlation focuses on the impact of inevitable bridge sudden damage on road network vulnerability.

### 2.1 Importance and exposure

### 2.1.1 Importance study

At present, the key aspect of vulnerability research is importance research. The main purpose of importance measurement is to compare and sort different elements. For the road network, elements refer to the links between nodes. The destruction of these elements represents the worst case, that is, the interruption or destruction of the link,
resulting in the congestion of the road network and the reduction of the transportation efficiency. These elements can also be regarded as the potential targets for counterattack against the system. Being able to identify important elements means that the targeted measures can be taken to reduce the risk of interference at these locations, or to provide the secondary optimal path between OD pairs for the road network that has been interrupted. The concept of element importance requires explicit or implicit assumptions on other scene dimensions, including the occurrence time, the duration, the degree of performance degradation, and etc., to make the comparison between elements meaningful. Therefore, the importance of an element can be calculated as the total impact of an interruption scheme involving the element, and some values of other dimensions can be used as conditions.

### 2.1.2 Exposure study

Vulnerability can be viewed from two different perspectives. The first is to pay attention to the social aspect of the system. For a specific individual, users often want to know how individual experience is affected under various possible interference situations, and what is the probability of each scenario? In addition, what is the impact of the worst-case scenario and the impact of long-term system outage?
After the concept of exposure is put forward, the vulnerability of road network can be better evaluated by combining exposure with scenario probability. From a social perspective, exposure can also be called conditional vulnerability. The concept of exposure is related to a variety of factors, such as geography, economy, population and other variables will cause the interruption of the road network, and the impact on specific users is different, that is, the exposure is different.

### 2.2 Correlation study

Importance study considers the importance of all components in the road network, and the importance degree of each component can be obtained by sorting the importance parameters of these components. The correlation is more focused on a component, and is the impact of the component on the road network. If the correlation degree is large, it means that the component has a greater impact on the road network. On the contrary, it shows that the component has little influence on the road network.
In fact, there is not much work on quantitative analysis of correlation in road network vulnerability analysis in the literature. However, with industrialization process, our construction capacity has long been in the forefront of the world, and different cities are also expanding rapidly, which brings the practical problem of increasing urban traffic pressure. In order to deal with this situation, the demand for urban bridge construction represented by urban viaduct is also increasing. Therefore, the urban road network in China will inevitably include bridges, so in the vulnerability analysis of urban road network in China, the correlation analysis of bridges is essential.
In this paper, the bridge correlation is divided into two parts. First part is the degree of correlation between the bridge and the whole road network. The second part is the connection degree between the bridge and other links in the road network. For the first part, it is needed to put forward the measurement index corresponding to the correlation, recalculate the influence degree of bridge in the road network, and
compare with the influence degree of other links in the road network; For the second part, it is needed to establish the corresponding measurement index, and get the correlation parameters through calculation, so as to determine the influence degree of each link in the road network on the bridge.

## 3 Modeling analysis

### 3.1 Importance model analysis

The idea of establishing the importance model is to turn the road network model into an organic combination of links and nodes. The importance parameters of each node and link are calculated by using the proposed calculation formula, and the importance degree of each node and link can be obtained by sorting them in a certain order. So the key steps of the problem can be divided into the following: (1) Road network modeling (2) Interrupt scene representation (3) Importance measurement. Finally, the corresponding calculation program is given.

### 3.1.1 Road network modeling: original method

The original method is one-to-one correspondence between the actual road network and the road network model, which can directly reflect the actual situation of the road network. It turns the road model in the road network into links, and the road intersection into road network nodes, so as to get the model diagram of the actual road network. The road network is modeled as an undirected graph. There are several nodes and links in the road network, and each network link $K$ has a fixed length and user travel time $t_{k}$. A group of special start/end nodes (OD pairs) are selected as the main start/end nodes. In the road network, every node and link are numbered. At the same time, the road network is divided into geographical regions according to the location of OD nodes. The original road network modeling example is shown in Fig. 1.


Fig. 1 Example of original road network modeling. Note: A1~D4 mean the start and end nodes of path, usually are important transportation nodes; k1~k21 mean path

### 3.1.2 Interrupt scenario representation

It is assumed that the interruption scenario consists of one or several links completely closed within a certain duration. During this period, the travel demand has no elasticity to the interruption. During the interruption period, the travel demand per unit time between each OD pair $(i, j)$, which can be expressed as $\mathrm{x}_{\mathrm{ij}}$. It is assumed that the change of travel compensation related to interruption is proportional to the increase of travel time or the duration of travel delay, that is, the delay of travel. Interruption scenarios can be defined by closing one or more links and nodes, which can be divided into two cases: single link interruption and area coverage network interruption.
(1) Single link interruption

Single link interruption can be applied to the situation that a certain road in the road network is interrupted and damaged, such as a traffic accident on a certain road, a certain road needs temporary repair and so on. Large scale network mainly adopts the way of single link interruption. For single link interruption, because the road network model adopts the undirected graph model, it is necessary to consider two directions of closing two-way links at the same time, which are represented by two opposite links.
(2) Area coverage interruption

For area coverage network interruption, the method proposed by Jenelius and Mattson (2012) is proper. The study area is completely covered by uniformly shifted meshes of uniform shape and size elements. The example diagram is shown in Fig. 2. Each cell represents the precise spatial location and range of the outage time. In order to simulate the event, any link that intersects could be partially or completely, with the unit is completely closed during the outage period, while all links that do not intersect with the unit are not affected at all. Therefore, the element


Fig. 2 Example of area coverage network interruption
$e$ contains all the links intersecting with a specific unit, the set $\varepsilon$ contains the elements corresponding to each grid cell intersected in the study area.

### 3.1.3 Importance measurement

The main purpose of importance measurement is to calculate the importance parameters of different elements (nodes and links), and then compare and sort them to get the importance degree of elements. In order to calculate the interference effect, the corresponding calculation model is established. Once shutdown occurs, the user will know the shortest path and travel time of empty scene and interrupt scene, as well as the duration of shutdown, expressed as $\tau$. If there are other shortest paths, the user can choose a new shortest path form, or wait until the link in element $e$ is reopened. How the user chooses depends on the length of time of the two methods. The travel time difference between the new shortest path and the original shortest path can be expressed as $\Delta t_{i j}^{e}(\tau)$. If there is no other shortest path $\Delta \mathrm{t}_{\mathrm{ij}}^{\mathrm{e}}(\tau)=\infty$, indicates that the user will wait until the link is reopened. Assuming that the travel time is evenly or randomly distributed over time, the total delay during the shutdown period $\Delta T_{i j}^{e}(\tau)$ is calculated as:

$$
\Delta T_{\mathrm{ij}}^{\mathrm{e}}(\tau)= \begin{cases}\mathrm{x}_{\mathrm{ij}} \Delta \mathrm{t}_{\mathrm{ij}}^{\mathrm{e}}\left(\tau-\frac{\Delta \mathrm{t}_{\mathrm{i}}^{\mathrm{e}}}{2}\right) & \Delta \mathrm{t}_{\mathrm{ij}}^{\mathrm{e}}<\tau  \tag{1}\\ \frac{\mathrm{x}_{\mathrm{ij}} \tau^{2}}{2} & \Delta t_{\mathrm{ij}}^{\mathrm{e}}<\tau\end{cases}
$$

where $x_{\mathrm{ij}}$ represents diapaus's travel demand per unit time between $i$ and $j$.
The overall importance of element $e$ is achieved by pairing all OD pairs' together $\Delta \mathrm{T}_{\mathrm{ij}}^{\mathrm{e}}(\tau)$. The results are as follows:

$$
\begin{equation*}
\mathrm{I}(\mathrm{e} \mid \tau)=\sum_{\mathrm{i} \in \mathrm{od}} \sum_{\mathrm{j} \epsilon \mathrm{od}} \Delta \mathrm{~T}_{\mathrm{ij}}^{\mathrm{e}}(\tau) \tag{2}
\end{equation*}
$$

The most direct method to calculate the impact of all OD on the design and scheme $\Delta \mathrm{T}_{\mathrm{ij}}^{\mathrm{e}}(\tau)$ is to calculate the travel time between all OD pairs in the empty scene $\mathrm{t}_{\mathrm{ij}}^{0}$. Then the element $e$ is closed and the link outage travel time $\mathrm{t}_{\mathrm{ij}}^{\mathrm{e}}$ of all OD pairs is calculated, the difference in travel time between the two $\Delta \mathrm{t}_{\mathrm{ij}}^{\mathrm{e}}(\tau)$ can be used as a representation of delay. For each element $e \epsilon \varepsilon$ repeat the process.

### 3.2 Correlation model analysis

### 3.2.1 Road network modeling: dual method

In the dual method, the actual road is transformed into a node in the road network model, and the road in intersected into a link. The example diagram of road network modeling in the dual method is shown in Fig. 3. Compared with the original method, the dual method cannot directly reflect the characteristics of the actual road network, but it is a secondary modeling based on the original method. The biggest advantage is that it can highlight the characteristics of the research object. For example, the main research object here is the road, and the evaluation objects of the subsequent correlation evaluation indexes are nodes, so it is more intuitive to transform road into node than link. In addition, for some complex networks, the use of dual method can reduce the number of nodes and links, so that the road network is simpler and the calculation process is more convenient.


Fig. 3 Example of dual road network modeling. Note: the number from 1 to 21 mean path

### 3.2.2 Measurement index

The correlation analysis mainly includes two parts, namely, the correlation degree between the bridge and the road network and the correlation degree between the bridge and other links in the road network. The analysis ideas of the two parts are to establish the corresponding measurement index first, and then integrate it into the evaluation system through the integrated method. The first part can be divided into three parts: node degree, proximity centrality and intermediary centrality, while the second part is mainly to establish interrupt centrality index to measure.

## (1) Node degree

In the road network model, node degree refers to the number of all links directly connected to a node, which can directly reflect the complexity of the connection between nodes and links, and then show the degree of association of the node in the road network. The greater the node degree of a node, the more connections it has with the surrounding links, and the greater the degree of association it has with the road network. The calculation formula of node degree is as follows:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{i}}=\sum_{\mathrm{j} \in \mathrm{~N}(\mathrm{i})} \mathrm{v}_{\mathrm{ij}} \tag{3}
\end{equation*}
$$

where $\mathrm{K}_{\mathrm{i}}$ represents the node degree of node $i$; $N(i)$ represents all nodes adjacent to node $i ; \nu_{\mathrm{ij}}$ represents the link between nodes $i$ and $j$.
(2) Approaching centrality

Proximity centrality refers to the reciprocal of the sum of the shortest distances from a node to all other nodes in the network. There is a shortest path between two nodes, and there is even more than one shortest path in the road network with
path weights of 1 . The shortest distance from a node to all other nodes can reflect the central position of the node in the road network to a certain extent, that is, the degree of association with the road network. The reciprocal of the sum of the shortest distances is used to express the proximity centrality. Therefore, the greater the proximity centrality, the smaller the sum of the shortest distances, and the greater the degree of association of the node in the road network. The calculation formula of proximity centrality is as follows:

$$
\begin{equation*}
C_{i}=\frac{1}{\sum_{j=1}^{N} d(i, j)} \tag{4}
\end{equation*}
$$

where $\mathrm{C}_{\mathrm{i}}$ represents the proximity centrality of node $i$; $N$ Represents the number of nodes in the road network; $d(i, j)$ represents the shortest distance between nodes $i$ and $j$.
In this paper, for the path weights are all 1 , and the dual method is used to model the road network, the shortest distance between two nodes should be expressed by the number of all nodes that the shortest path of two nodes passes through.
(3) Intermediary centrality

The intermediary centrality of a node refers to the number of nodes passing through the shortest path between all nodes in the road network. The higher the intermediary centrality of a node, the more the number of shortest paths that the node can affect, and the greater the impact on the whole road network. In order to measure the intermediary centrality of different nodes, the ratio of the number of shortest paths passing through a node to the number of all shortest paths between two nodes is used to calculate the intermediary centrality of the node:

$$
\begin{equation*}
C_{B_{i}}=\sum_{j}^{N} \sum_{k}^{N} \frac{g_{j k}(i)}{g_{j k}}, i \neq j \neq k, j<k \tag{5}
\end{equation*}
$$

where $\mathrm{C}_{\mathrm{B}_{\mathrm{i}}}$ represents the intermediary centrality of node $i ; \mathrm{g}_{\mathrm{jk}}$ represents the number of shortest paths between nodes $j$ and $k ; \mathbf{g}_{j \mathbf{k}}(\mathrm{i})$ represents the number of node $i$ in all shortest paths between nodes $j$ and $k$.
Similarly, for the usage of the dual method to model the road network, the shortest path should be understood as the path with the least number of nodes. In the road network modeled by dual method, the intermediary centrality of nodes can directly reflect the influence degree of a certain road in the actual road network. The more times the shortest path passes through the road, the greater the hub conversion effect of the road in the whole road network, and the greater the correlation degree between that road and the road network.
(4) Interrupt centrality

Just a node $i$, the shortest path between the other two nodes $k$ and $l$ in the road network may pass through $i$ and another node $j$, or may not pass through $i$ and $j$, or may pass through one of $i$ and when the node $j$ is closed, if node $j$ is not on the shortest path of nodes $k$ and $l$, the shortest path will not change. On the contrary,
the shortest path of node $k$ and $l$ will certainly change, which will produce three results for node $i$, that is, the traffic flow through node $i$ will not change or increase or decrease. The invariable case is that the original shortest path and the new shortest path between nodes $k$ and $l$ do not pass through node $i$, or the original shortest path and the new shortest path both pass through node $i$ and the number is not changed. In addition, the number of nodes passing through $i$ in the new shortest path between nodes $k$ and $l$ is more than that in the original shortest path. In the case of reduction, the number of the new shortest path between nodes $k$ and $l$ passing through node $i$ is less than that of the original shortest path, which reduces the traffic flow passing through node $i$. The calculation formula is as follows:

$$
\begin{equation*}
C_{D_{i-j}}=\sum_{j}^{N} \sum_{k}^{N} \sum_{l}^{N}\left|\frac{g_{k l-j}(i)}{g_{k l-j}}-\frac{g_{k l}(i)}{g_{k l}}\right|, i \neq j \neq k \neq l, k<1 \tag{6}
\end{equation*}
$$

where $C_{D_{i-j}}$ represents the interrupt centrality of node $i$ to node $j ; \mathrm{g}_{\mathrm{kl}}$ represents the number of shortest paths between nodes $k$ and $l ; \mathrm{g}_{\mathrm{kj}}(\mathrm{i})$ represents the number of shortest paths between nodes $k$ and $l$ passing through node $i ; \mathrm{g}_{\mathrm{kl}-\mathrm{j}}$ represents the number of shortest paths between node $k$ and $l$ after node $j$ is closed; $\mathrm{g}_{\mathrm{kl}-\mathrm{j}}(\mathrm{i})$ represents the number of shortest paths between node $k$ and $l$ passing through node $i$ after node $j$ is closed.
It can be seen from the calculation formula that if $g_{k l-j}(i) / g_{k l-j}-g_{k l}(i) / g_{k l}$ is less than 0 , it means that the closing of node $j$ will reduce the traffic flow of node $i$, which can be considered as a favorable impact. On the contrary, if it is greater than 0 , it means that the closing of node $j$ will cause the increase of traffic flow of node $i$, which can be considered as an adverse effect.

### 3.2.3 Grey relational analysis

The comprehensive evaluation method is using multiple indicators to evaluate an eigenvalue. The calculation method of eigenvalue needs to be determined, according to the definition of multiple indicators. Multiple indicators are converted into an eigenvalue, and then are calculated and sorted, in order to obtain the influence degree of each factor on the whole.
The Grey correlation analysis method is suitable for measuring the correlation degree of multiple factors, and its core element is the correlation degree. It is to calculate the correlation degree between different factors, and then sort them to get the correlation degree between different elements, that is, the correlation need to be analyzed. In this paper, the two different factors are the bridge with other links and the overall road network. Based on the evaluation index established above, the comparison sequence and reference sequence of each node are obtained, and then the dimensionless treatment and each index weight are given to calculate the correlation degree between the two elements. Subsequently, the correlation degree of the bridge relative to other links in the road network is obtained.

The Grey correlation analysis method can be used to calculate the correlation degree between all nodes in the road network and the overall road network. As for the second
part of the correlation analysis, the measure of interruption centrality proposed above can be used to calculate.

## 4 Numerical examples

In this section, an area in Shanghai is selected to conduct modeling analysis according to the above methods to verify the feasibility of the proposed model. As the actual data such as road traffic flow, road specific length, bridge traffic flow, road and bridge capacity are difficult to obtain, the relevant calculation data are simulated according to the relative value of road type, number of vehicles and link length. Therefore, the calculation results of numerical examples in this section cannot be directly applied to the actual road network. However, the main purpose of this paper is to verify the feasibility of the relevant models, which can be applied it to the actual road network in the future.

### 4.1 Importance example

### 4.1.1 Road network regional modeling (original method)

The road network area of the example on both sides of Yangpu Bridge is selected. The whole road network area includes Yangpu Bridge, main roads and secondary roads on the east and west banks. The main roads map of the actual road network map is shown in Fig. 4.

The original method is used to divide it into a network composed of five main roads and three secondary roads, resulting in 14 nodes represented by numbers $1-14$ and 19 links represented by k1-k19. According to the actual location of the traffic hub on the map, six OD pairs represented by A, B, C, D, E, F are determined. The grid divides the whole road network area into 15 small areas which are all $1000 \times 1000 \mathrm{~m}^{2}$. The original road network model diagram of each node and link number is shown in Fig. 5.


Fig. 4 Main roads map


Fig. 5 Road network model diagram (original method)

### 4.1.2 Data determination

The selected road network area is located in the city center, so it can be assumed that the duration of each link outage is 30 min . As for the traffic flow and travel demand of each link, it can be assumed that the traffic flow and travel demand of each link in the road network are equal, and the traffic flow of each link can be set as 30 vehicles per minute, and the travel demand can be set as 20 vehicles per minute.
In addition, the key data to be determined is the travel time of each link. The length of each link is different. The approximate length of each link is determined according to the map data. At the same time, due to the different demands of each road for users, the road congestion in the morning and evening peak is also different. For the sake of uniformity, the morning peak around $9 \mathrm{a} . \mathrm{m}$. is selected and the map is used to get the congestion of each link. The travel time of all links can be divided into four levels, namely 3 $\mathrm{min}, 2 \mathrm{~min}, 1.5 \mathrm{~min}$ and 1 min . The specific travel state of each link is shown in Table 1.

### 4.1.3 Calculation results and analysis

Under the above analysis and data setting, the calculation program is written in MATLAB software, which can calculate the importance of each link and area unit in the case of single link interruption and area coverage interruption. The calculation results of the importance parameter of each link in the case of single link interruption are shown in Table 2, and the calculation results of the importance parameter of each regional unit in the case of regional coverage interruption are shown in Table 3.
From the calculation results in Table 2, it can be concluded that link 18 is the most important link, followed by link 2 . The importance of links 6,9 and 12 is the lowest, even the calculation result is zero, which shows that these three links have no influence on the selection of the shortest path between OD pair nodes in the example network, that is, these three links are not on the shortest path between OD pair nodes.

Table 1 Link travel state

| Link number | Length / km | Congestion | Travel time / min |
| :---: | :---: | :---: | :---: |
| k1 | 2.21 | traffic jam | 3 |
| k2 | 1.32 | open | 1.5 |
| k3 | 1.20 | open | 1 |
| k4 | 0.94 | open | 1.5 |
| k5 | 1.13 | open | 1.5 |
| k6 | 1.33 | open | 2 |
| k7 | 1.86 | open | 2 |
| k8 | 2.19 | open | 1 |
| k9 | 0.95 | open | 2 |
| k10 | 1.41 | crowding | 1.5 |
| k11 | 1.30 | open | 2 |
| k12 | 1.08 | crowding | 1 |
| k13 | 1.04 | open | 1 |
| k14 | 0.99 | open | 1.5 |
| k15 | 0.87 | crowding | 1.5 |
| k16 | 0.90 | crowding | 1.5 |
| k17 | 1.31 | open | 1.5 |
| k18 | 1.32 | open | 1.5 |
| k19 | 1.23 | open | 1.5 |

The congestion and travel time is calculated according to Gaode Map. The red color means the most crowded, the orange color means the second crowded

Table 2 Importance parameter results of single link interruption

| Link | Importance parameter | Link | Importance parameter |
| :---: | :---: | :---: | :---: |
| k1 | 1755 | k11 | 8635 |
| k2 | 11655 | k12 | 0 |
| k3 | 8020 | k13 | 4600 |
| k4 | 8100 | k14 | 1775 |
| k5 | 5660 | k15 | 5790 |
| k6 | 0 | k16 | 3470 |
| k7 | 3500 | k17 | 11320 |
| k8 | 1755 | k18 | 21645 |
| k9 | 0 | k19 | 8070 |
| k10 | 3420 |  |  |

The red color means the highest score, which means k 18 is the most important path in the rode network; The green color means the lowest score, which means $\mathrm{k} 6 \backslash \mathrm{k} 9 \backslash \mathrm{k} 12$ are the least important path in the rode network

Table 3 Importance parameter results of coverage interruption

| Region | Importance parameter | Region | Importance parameter |
| :---: | :---: | :---: | :---: |
| 1 | 90000 | 9 | 9100 |
| 2 | 90000 | 10 | 1755 |
| 3 | 91180 | 11 | 4600 |
| 4 | 103230 | 12 | 1755 |
| 5 | 90000 | 13 | 25905 |
| 6 | 91180 | 14 | 0 |
| 7 | 11655 | 15 | 11320 |
| 8 | 11655 |  |  |

The red color means the highest score, which means region 4 is the most important region in the rode network; The green color means the lowest score, which means k6\k9\k12 are the least important region in the rode network

From the calculation results of the importance of each regional unit in the case of regional coverage interruption in Table 3, it is not difficult to see that the importance parameter of region 4 is the largest, which indicates that the importance of region 4 is the highest.The importance parameter of region 14 is the lowest, even zero, which indicates that the importance of region 14 is the lowest, and the closure of the area unit has no effect on the transportation efficiency of the whole road network.
The calculation results of the two cases can be verified with each other, indicating that the calculation model is indeed applicable to the two cases.

### 4.2 Correlation example

Similar to the importance example, the first step in the calculation process of the correlation example is to model the road network. In order to facilitate the comparative analysis of importance and correlation, and considering the similarities in definitions between them, the same road network area is used in the calculation of correlation example, but the modeling method is different. The second step is to calculate the corresponding indicators and complete the two parts of the calculation of correlation analysis. Finally, the results are compared and analyzed, and the conclusion corresponding to the road network is obtained.

### 4.2.1 Road network modeling (dual method)

The road network is modeled as nodes according to the dual method, and the nodes are connected according to the connectivity of the roads in the actual road network. The work is based on the original road network model diagram. The road network diagram modeled by the dual method is shown in Fig. 6.

### 4.2.2 Data calculation

According to the dual method road network model figure obtained in Sect. 4.2.1, the value of each measurement index of each node in the diagram is calculated as shown in Table 4. The correlation degree of each node can be calculated according to the calculation logic of Grey Correlation Analysis Method by using the calculated measurement indicators, as shown in the Fig. 7.


Fig. 6 Road network model diagram (dual method)

Table 4 Measurement index values

| node | Node degree | Approaching centrality | Intermediary centrality | Interrupt centrality | Influence |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3 | 0.0204 | 10.0000 | 1.8333 | - |
| 2 | 4 | 0.0238 | 34.3333 | / | 1 |
| 3 | 3 | 0.0213 | 9.0000 | 1.6000 | - |
| 4 | 3 | 0.0182 | 4.0000 | 0.0000 | 0 |
| 5 | 4 | 0.0233 | 21.5833 | 5.9167 | - |
| 6 | 4 | 0.0238 | 23.8333 | 8.0833 | - |
| 7 | 3 | 0.0196 | 5.7500 | 0.0833 | - |
| 8 | 4 | 0.0204 | 6.5000 | 1.0000 | 0 |
| 9 | 4 | 0.0213 | 7.1667 | 0.5000 | - |
| 10 | 3 | 0.0182 | 4.5000 | 0.000 | 0 |
| 11 | 4 | 0.0233 | 22.1667 | 6.1667 | + |
| 12 | 4 | 0.0238 | 19.8333 | 4.8333 | $+$ |
| 13 | 4 | 0.0196 | 10.1667 | 0.3333 | $+$ |
| 14 | 4 | 0.0213 | 7.1667 | 0.3333 | $+$ |
| 15 | 4 | 0.0238 | 23.5000 | 9.2500 | $+$ |
| 16 | 3 | 0.0196 | 6.4167 | 0.2500 | $+$ |
| 17 | 3 | 0.0204 | 11.0000 | 1.0000 | $+$ |
| 18 | 4 | 0.0238 | 34.1667 | 33.6667 | $+$ |
| 19 | 3 | 0.0213 | 8.6667 | 1.5000 | + |

" + " in the column of "Influence" indicates that the closure of the node will have adverse impact on the bridge node (No. 2),
"-" indicates that the impact is favorable, and " 0 " indicates that there is no impact

### 4.2.3 Calculation and results analysis

It can be seen from Fig. 7 that the correlation degree of node 2, that is, the bridge node, is the largest, reaching 1 , which indicates that the correlation between the bridge and the road network is the highest in the road network, and the operation of the bridge is always related to the transportation efficiency of the road network. The correlation


Fig. 7 Node correlation degree
degree of other nodes in the road network is between 0.6 and 0.85 , which is smaller than that of bridge nodes. Therefore, the conclusion can be drawn that in the road network, the bridge has the highest correlation with the overall road network.
As for the correlation analysis between the bridge and other nodes in the road network, it needs to be completed with the help of interrupt centrality. The interrupt centrality of each node and bridge is shown in the Fig. 8. It shows that node 18 has the highest interruption centrality to the bridge and is much higher than the interruption centrality of other nodes to the bridge nodes, which indicates that node 18 has the highest correlation with the bridge nodes.

## 5 Conclusions

This paper expounds the importance and correlation of quantitative indicators of urban bridges and roads in the vulnerability analysis of road network, establishes the corresponding mathematical model, calculates the quantitative indicators, and evaluates the vulnerability of road network. The main contributions and findings are as follows:
(1) From the definition of road network vulnerability, this paper further expounds the concepts of importance and correlation in the road network vulnerability analysis,.


Fig. 8 Node interrupt centrality
and determines the completion route of vulnerability analysis by separating the concept of importance and correlation into two parts.
(2) For the importance analysis part, the road network modeling method is used and the mathematical model of road importance analysis is established. Numerical analysis of the road importance is applied to the selected areas on both sides of Yangpu Bridge, Shanghai, it turns out that the mathematical model is applicable to importance valuation.
(3) For the correlation analysis part, the mathematical analysis model is established by defining multiple measurement indexes, and applied to the same scene to evaluate the correlation between important nodes (bridge) in the road network and other nodes as well as the whole road network. The results show that the model established in this paper can be applied to the future planning and health monitoring of road network.

However, the current research on the vulnerability of road networks is only limited to the stage of simple road network models. The road network area discussed in this paper is relatively small, and the actual road network calculated is relatively small in both node and link numbers. Moreover, the intersection and overlap between links are relatively simple, making it difficult to directly apply to large-scale road networks in actual cities. At the same time, due to insufficient acquisition of actual travel data, the calculation data is basically estimated based on certain map information, and there is room for improvement in the accuracy of the calculation results.

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## Authors' contributions

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## Declarations

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