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Polya urn model-based performance assessment of PSC bridges: prestress loss consideration

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Abstract

A methodology for performance assessment of prestressed concrete (PSC) girder bridge system, based on strain monitoring in limited number of girders, is proposed in this paper. The methodology uses Polya urn model for determining probabilities of the bridge system being in different condition states with respect to loss of prestress. Performability measure is used for describing the performance of the bridge system. A condition state is assigned for the bridge system from a predefined set of condition states. The time for detailed inspection is determined as the time instant at which the performability of the bridge system becomes less than the target/required performance level. Performance assessment of a bridge system with one hundred PSC girders is considered for illustrating the methodology. The obtained values of condition state probabilities and performability for the considered scenarios (i.e., different number of monitored girders with prestress loss exceeding the allowable value) suggest that the methodology is able to consider the value of available information.

Keywords: Prestressed concrete bridge system, Polya urn model, Prestress loss, Reward rate, Performability

1 An explanation of why your manuscript should be published in advances in bridge engineering

- For the first time the authors propose the Polya urn model-based methodology for assessing the performance of prestressed concrete (PSC) girder bridge system, using strain monitoring data from a limited number of girders.
- The methodology is useful when the monitored girders are randomly located in the bridge system. The methodology is illustrated considering a PSC bridge with hundred girders, with 20 PSC girders instrumented for strain monitoring, and computing the performability of the system considering estimated loss of prestress from the monitored girders. Estimating the performability of the system will be help in decision making with respect to detailed bridge inspection.
- The methodology belongs to the domain of data-driven approach

2 Introduction

Several prestressed concrete (PSC) girder bridge systems have large number of girders which are nominally similar and are exposed to almost similar environment along the length of the bridge system. For instance, in Chennai, India, the Mass Rapid Transit System is having a PSC bridge system with 288 nominally similar girders each of 18.0 m span and 270 girders each of 22.5 m span along the length. Here after, in this paper, such bridge systems are simply referred to as bridge system. During the service life, typical bridge system, depending on traffic, environmental conditions, and inspection and maintenance strategy adopted, can be subjected to different degradation mechanisms. One of the important mechanisms of degradation in performance of PSC girders is the prestress loss-induced excessive deflection of bridge span. The time-dependent prestress losses, during the service life, due to relaxation in prestressing steel and shrinkage- and creep- of concrete may result in excessive deflections and associated serviceability issues in bridge systems. For instance, within 18 years, an average prestress loss of 50% was reported for the Koror-Babeldaob Bridge in Palau, which has lead to excessive camber and required remedial measures (Bazant et al. 2011). Numbers of cases wherein measured deflections in PSC bridge girders exceed the allowable deflections have been reported by Bazant et al. (Bazant et al. 2011). This shows the need for periodic inspection and maintenance of the PSC bridge girders for ensuring the functionality and for service life extension.

In general, the inspections carried out on bridges can be classified as: (i) General/routine inspection, (ii) Detailed inspection and (iii) Special inspection (Handbook on inspection of bridges – RDSO). The national and state highways departments maintain the inspection reports of the bridges falling in their respective purview. These reports help in assessing the condition of bridges. Depending on the computed condition rating of a bridge, its remaining life or life expectancy can be estimated.

Monitoring the loss of prestress in all of the PSC girders in a bridge system of the type considered in this study may not be economically feasible and/or practical. Also, the funds available for bridge inspection and maintenance are limited, and hence it is essential to develop strategies which are more rational and optimal for scheduling of inspection and maintenance activities (Sánchez-Silva et al. 2016; Schöbi and Chatzi 2016).

A comprehensive study presenting the importance of consideration of uncertainties and probabilistic modelling in assessment of life expectancy of bridges and other highway assets have been presented in NCHRP report (NCHRP 2012). The estimated expected lives can be used, amongst others, for prioritization of projects. One of the approaches mentioned in this report is Markov-based duration model.

The methodologies, in general, used to assess the condition rating of bridges can be classified as follows:

- Empirical modelling approaches (Agrawal and Kawaguchi 2009; Lavrenz et al. 2015; Moomen et al. 2016; Saeed et al. 2017a; Saeed et al. 2017b; Tolliver and Pan 2011; Van Noortwijk and Klatter 2004; Veshosky et al. 1994; Zhang et al. 2003).
- Markov Chains (Balaji Rao and Appa Rao 1999; Amin and Adey 2015; Estes and Frangopol 2001; Hallberg 2005; Jiang and Sinha 1989; Morcous 2006; Robelin and Madanat 2007; Sobanjo 2011).

- Discrete choice models (Saeed et al. 2016; Saeed et al. 2017c).
- Machine learning techniques such as ANN, instance-based learning, inductive learning (Melhem and Cheng 2003; Narasinghe et al. 2006).
- Duration models with exponential, Gamma, Weibull, or Rayleigh distributions (Klatter and Van Noortwijk 2003; Hearn and Xi 2007; Nicolai 2008; Ng and Moses 1996).

In the present study, the focus is on prestress loss estimation due to ageing effects alone (effects due to cracking and spalling and possible breakage of prestressing tendon are not considered) and based on strain data obtained during inspection or continuous monitoring of limited number of PSC girders in a bridge system. As pointed out in NCHRP report (NCHRP 2012), there is a need to account for appropriate uncertainties and the quantity of data available.

This can be realised by developing methodologies for prestress loss assessment in the PSC girders based on monitoring data from few girders in the bridge system (Saliya 2008). For condition assessment of bridge system using information from limited inspection, Markov chain (MC) models have been proposed in the literature. However, the use of MC models is limited by the requirement of availability of inspection data on adjacent girders. Balaji Rao and Anoop (Balaji Rao and Anoop 2019) proposed a Polya urn model-based procedure for prestress loss assessment of girders in a PSC bridge system, considering monitored strain data from few numbers of nominally similar girders. The proposed procedure integrates a stochastic pure birth process model (namely, the Polya urn model) with the AASHTO LRFD 2012 prestress loss estimation method (AASHTO 2012). The procedure relaxes the requirement for adjacency of the girders to be monitored needed for the Markov chain model, and hence is more flexible.

In the present study, a methodology for performance-based condition assessment of a bridge system, based on prestress loss estimated from the strains obtained from limited number of prestressed concrete (PSC) girders, is proposed. The condition state probabilities are determined using the Polya urn based model (Balaji Rao and Anoop 2019). Performability measure is used for describing the performance of the bridge system. The methodology makes use of a data-driven approach. The performance assessment procedure presented in this paper can be used as a stand-alone module (if the data is obtained from inspections) or can be integrated in to an online decision making module if the data is acquired in real time (viz. continuous structural health monitoring). The latter can be used for quick assessment of the condition of the bridge stock with respect to the target performance requirements. The methodology is illustrated by considering a simple example of a PSC bridge system with one hundred girders. It is noted that some of the information presented in this paper are based on Ref. (Balaji Rao and Anoop 2019). The aim is to reach wider readership.

3 Why data-driven approach?

The structural health monitoring (SHM) has traditionally been using a model-based approach, however, the complexities involved in modelling the structure has made data-driven approaches to SHM an attractive alternate (Sen and Nagarajaiah 2018). As pointed out by Hughes et al. (Hughes et al. 2021), a decision maker is tasked with activities related to making decisions that are robust enough to handle the uncertainties in the data regarding the damage state of the structure. The authors recommend a probabilistic risk-based decision approach for addressing this complex SHM problem (more so when the decisions are made with respect to populations of structures). Ye et al. (Ye et al. 2019) deal with issues related to creation of digital twin of bridges. They recommend data-driven approach for updating the relevant models. Statistical learning algorithms are widely employed in data-driven approaches for SHM. The statistical learning theory, as stated by Bousquet et al. (Bousquet et al. 2004), provides a framework for studying the problem of inference, that is of gaining knowledge, constructing models, making predictions or making decisions using a set of data. In data-driven approaches, the aim is the development of a statistical representation of the system. According to Worden and Manson (Worden and Manson 2006), data-driven approaches provide a natural framework for addressing the first three of the four hierarchical levels (namely, detection, localisation, assessment and prediction) of SHM.

A statistical reinforced learning model using Polya urn model (Morcrette 2012) is considered in this study for developing the methodology for performance-based condition assessment of prestressed concrete bridge system. The methodology can work with limited amount of monitoring data and can provide an effective framework for online SHM. The details of the model are presented in the following section. The data obtained from the inspection or on-line monitoring is assumed to have been filtered for the various noises and varying unknown conditions using techniques such as those presented by Figueiredo et al. (Figueiredo et al. 2014) and Liu et al. (Liu et al. 2019).

4 Methodology for performance assessment of bridge system

A procedure based on Polya urn model has been proposed by Balaji Rao and Anoop (Balaji Rao and Anoop 2019) for prestress loss assessment in girders of a PSC bridge system, using monitored strain data obtained from a limited number of girders. In the present study, the procedure is further extended for performance-based condition assessment of girders in a PSC bridge system. The steps involved in the proposed methodology are as follows.

Step 1: Determination of the expected loss of prestress at different times: In this study, the loss of prestress due to shrinkage, creep and relaxation, and the total prestress loss at different times are predicted using the detailed method specified in AASHTO LRFD 2012 (AASHTO 2012).

Step 2: Estimation of the actual loss of prestress based on monitored strain data: There exist different methods for determining the loss of prestress in PSC girders of an existing bridge system. One method is to use the monitored strain data. Another method is to use local elastic stress relief techniques such as concrete core trepanning technique, centre hole stress relief technique, concrete stress relief core technique, steel stress relief hole technique (Kesavan et al. 2005). In the present study, for the purpose of illustrating the proposed methodology, actual prestress losses in the monitored PSC girders are estimated using the strain in concrete at the level of centre of gravity of prestressing steel monitored using embedded strain gauges.

Step 3: Determination of the condition state probabilities for the bridge system using Polya Urn model: The condition states of the bridge system is defined, in this paper, by the specified percentage of bridge girders having loss of prestress more than allowable value. Using the Polya urn model, the probabilities of bridge system in different condition states are estimated.

Step 4: Computation of performability of the bridge system: In the present study, performability is used as the measure for carrying out performance-based condition assessment of the bridge system.

The details related to these steps are given in the following sections.

4.1 Determination of allowable loss of Prestress at different times

AASHTO LRFD 2012 (AASHTO 2012) provides a 'detailed prestress loss estimation method' for the estimation of time-dependent prestress losses in the design of PSC girders (Swartz et al. 2012). The information about the concrete mixture proportions is not required for the use of this method. The prestress loss provisions of AASHTO LRFD 2012 have been evaluated by Garber et al. (Garber et al. 2016), using an extensive database. The database contained details of 237 girders representative of the actual PSC bridge girders. From the study, it is found that the ratio of estimated (using the detailed AASHTO LRFD procedure)-to-measured total prestress loss has a mean value of 1.25 and a coefficient of variation of 0.24 (Garber et al. 2016). While these statistics are useful for determining the characteristic value of the stated ratio, it has been found that for as many as 30 girders in the database created by Garber (Garber 2014), the predictions by AASHTO 2012 are nonconservative and hence in the present study, the mean value only has been used. Since the total prestress losses predicted at different times would be considered in the design, these values are taken to be the allowable prestress loss ($P_{L\_all}(t)$).

4.2 Estimation of actual loss of Prestress using monitored strain data

For PSC bridge girders it is found that the monitoring of prestress loss using strain sensors (Cousins 2005; Barr et al. 2009; Yang and Myers 2005; Roller et al. 2011) is more useful than the other methods (ACI 423 2016). For PSC bridge girders, use of strain sensors for health monitoring is found to be more suitable due to the following advantages: (i) accurate and stable long-term measurements, (ii) no requirement of calibration for every structure, and, (iii) direct relation of the measured strains to prestress losses (Abdel-Jaber and Glisic 2018). The strain in concrete at the level of centre of gravity of the prestressing steel can also be continuously monitored using embedded strain gauges (Ravisankar et al. 2008; Kamatchi et al. 2014). In this case, it is assumed that just after the transfer of prestress, the strain is measured which is considered as the baseline value, $\epsilon_{\text{CG, baseline}}$. At any time t after transfer of prestress, the loss of prestress is given by:

$$\Delta \mathrm{f}_{\text{ps,estimated}}(\mathrm{t}) = E_{ps} \times \Delta\varepsilon_{\mathrm{CG}}(\mathrm{t}) \tag{1}$$

where

$$\Delta\varepsilon_{\mathrm{CG}}(\mathrm{t}) = \varepsilon_{\mathrm{CG}}(\mathrm{t}) - \epsilon_{\text{CG,baseline}} \tag{2}$$

$\varepsilon_{\mathrm{CG}}(\mathrm{t})$ is the measured strain at time t in concrete at the level of centre of gravity of the prestressing steel and E_{ps} is the modulus of elasticity of the prestresing steel.

The prestress loss (in percentage) is obtained as:

$$f_{psloss,predicted}(t) = \frac{\Delta f_{ps,estimated}(t)}{f_{st}} \times 100 \quad (3)$$

The predictions made based on Eq. (3) are considered to be satisfactory based on the results presented in Appendix 2.

The loss of prestress due to corrosion-induced cracking of concrete in the PSC bridge girders is not considered in the present study. It is noted that, Dai et al. (Dai et al. 2020) have reported that for corrosion losses less than 6.6%, the reduction in effective prestress due to corrosion-induced concrete cracking is negligible. However, for corrosion losses more than 6.6%, the prestress loss due to corrosion-induced cracking can be estimated using the model proposed by Dai et al. (Dai et al. 2020) and incorporation of the same in the methodology proposed in the present study needs further investigation and is not attempted in this paper.

4.3 Polya urn model for condition assessment of PSC girders

The Markov chain models discussed in Appendix 1 are more appropriate when the baseline information about the girder in the bridge system is available and these models make use of the one-step dependence with respect to index space for a girder and between two adjacent girders with respect to evaluation of condition state of the bridge system. These difficulties can be overcome by considering Polya urn model, which is a baseline-free model and data-driven model with respect to structural health monitoring terminology (Worden and Manson 2006).

Polya urn models have been widely considered in classical statistics. Its applications can be found in sciences, economics etc. due to the impetus given to the data driven approaches for decision making (Pasanisi (Pasanisi 2014) and the references therein). In structural engineering its potential for making engineering decisions is yet to be explored.

The recent developments in sensor technology, IoT etc. have enabled remote structural health monitoring (RSHM) of bridges. Using RSHM bridge-specific relevant data can be acquired at a central place for making asset management decisions. While considerable progress has been achieved in RSHM, there is a scope for development of methodologies for making engineering decisions at macroscopic level, using the data obtained during RSHM, for asset management. The need for development of such methodologies has been brought out by Omar and Nehdi (Omar and Nehdi 2018). In this paper the possibility of development of Polya urn model is explored.

More details of the model are presented in Balaji Rao and Anoop (Balaji Rao and Anoop 2019). The details, as required for the present paper, are presented below.

It is assumed that the environment to which the girders in the PSC bridge system are exposed is nominally similar. This assumption is valid for number of bridge systems. For instance, in Chennai, India, the Mass Rapid Transit System with 288 nominally similar PSC girders each of 18.0 m span and 270 girders each of 22.5 m span, is located along the coastline of Chennai, and is exposed to a more or less similar environment.

A small number of PSC girders are assumed to have been instrumented with strain gages on concrete surface at the level of centre of gravity of the prestressing steel. Using monitored strain data, the loss of prestress in these girders can be estimated.

The monitored or measured strains can come from health monitoring or measured during inspection on limited number of girders. Based on the loss of prestress estimated in these monitored girders, the interest is in knowing the percentage, p, of girders with loss of prestress more than the allowable in the bridge system. In the present study, this is achieved using the Polya urn model. The quantity p can be used for assessing the condition state of the bridge system.

Polya urn model is a statistical model, and the urn process associated is a type of pure birth process. Polya urn represents a system having two types of objects called the balls. Different ball colours are used for distinguishing the object types. The PSC girders are the objects (or balls) in the present study. The state of loss of prestress is indicated by the colour of the ball, i.e., if the colour of the ball is black, then the loss of prestress in the girder is less than the allowable. If the colour of ball is white, then the loss of prestress is more than the allowable. The state of loss of prestress in the monitored PSC girders is represented by the initial configuration of the Polya urn. In a basic trial (or step), one randomly selects a ball from the urn, and subsequently returns it along with an identical ball (i.e., with the same color) to the urn. Each outcome of the trial of picking a ball from the urn is equivalent to the inspection of one of the PSC girders, which was not initially considered for monitoring. That is, based on the colour of the selected ball, an inference on the state of loss of prestress in each of the girders not monitored is made. It stands to reason that there is a higher likelihood for the PSC girder to be inspected next has loss of prestress greater than the allowable is higher, if, the loss of prestress is greater than the allowable in more number of monitored girders. This reasoning, also called the self-reinforcing property) is in-built in the Polya urn model (Pasanisi 2014).

Let B and W be the numbers of black- and white- balls, respectively, present in the urn initially. Then (B, W) is the initial configuration of the Polya urn and the total number of balls is $n = \mathrm{B} + \mathrm{W}$. After a number of steps (or trials), say N ($\mathrm{N} = \mathrm{n} + \mathrm{M}$) is the number of balls available in the urn, where M is the number of steps. After M trials, the probability of having exactly p% of the balls as white, is given by (Mahmoud 2000; Antal et al. 2010):

$$\mathrm{Pr}_{\mathrm{W_M}} = \binom{\mathrm{B_M}-1}{\mathrm{B}-1}\binom{\mathrm{W_M}-1}{\mathrm{W}-1}\binom{\mathrm{N}-1}{\mathrm{B}+\mathrm{W}-1}^{-1} \tag{4}$$

where $W_M = \frac{p}{100}N$, $B_M = N - W_M$. B_M and W_M should be integers.

The probability that up to m balls are white in the urn ($\mathrm{PR_m}$) is:

$$\mathrm{PR_m} = \mathrm{P}(\mathrm{W_M} \le \mathrm{m}) = \sum_{\mathrm{k}=1}^{\mathrm{m}} \mathrm{Pr_k} \tag{5}$$

where $\mathrm{Pr_k}$ is computed using Eq. 4 by substituting $\mathrm{W_M}$ with k.

In this study, the condition state (CS(t)) of the PSC bridge system at time t is defined on the basis of the percentage of PSC girders with loss of prestress higher than the allowable as Very Good (0%–20%), Satisfactory (20%–40%), Poor (40%–60%), Serious (60%–80%), and Critical (80%–100%). Since the white ball indicates that in the PSC girder, the loss of prestress is more than the allowable, the probability distribution of percentage of girders in the PSC bridge system with loss of prestress more than allowable can be determined using Eq. 5. From this probability distribution, the probabilities ($p_i(t)$, $i = 1, \ldots, 5$) that the bridge system being in different condition states at time t are determined.

4.4 Performability

For performance-based condition assessment of the PSC girder, the condition state need to be related to the performance requirements. The usefulness of performability analysis for evaluating the performance of PSC girders under service loads is being explored in this paper.

Performability is the ability of the system to perform reliably even under degrading conditions and available to the engineer over intended life. From this view point, the tools required to evaluate the performability are reliability and availability analyses, method for estimation of response of the system suspended in a given environment. The uncertainties arising at different levels of evaluation should be given due consideration. In order to evaluate performability the following are needed: (i) a measure of performability, (ii) considerations and modelling of performability, and (iii) methodology for performability analysis.

The performability measure combines both the reliability and performance measures (Sanders and Meyer 1991). Platis (Platis 2006) proposed a new performability measure, within the framework of Markov chains (homogeneous or non-homogeneous, depending on type of problem considered), to account for the transition of the system from one state to another through instantaneous or impulse rewards and by classical or cumulative performability measure for the system spending in a given state over period of time.

Performability measure as proposed by Platis (Platis 2006) is an improvement to the availability measure for highly available systems (those systems whose availability and reliable performance are very vital at certain important times). The infrastructural facilities, such as bridge systems (considered in the present study), can be classified as highly available systems as they are supposed to meet the target reliability requirements under the service loads during their life.

Performability models have been introduced by Beaudry (Beaudry 1978) by defining the combined measures of performance and reliability. In general, performability models consist of a stochastic process which will describe the system evolution and a reward structure which will relate the possible system behaviour to a specified performance variable. While the performance variable can be instant-of-time variable, interval-of-time variable or the time-averaged interval-of-time variable (Sanders and Meyer 1991), the last two performance variables are not very sensitive to the system state at a particular instant of time as compared to the first one, especially at large times (Smith et al. 1988). Hence, for making decisions with respect to inspection, repair or replacement based upon the system state at a given instant of time, the instantaneous reward will be more useful. However, for selecting the best alternative based on a comparison of performances over a period of time for different alternatives, the accumulated- and time-averaged accumulated- rewards will be useful. A class of models that are increasingly used in the estimation of system performance is the performability model (Platis 2006).

Meyer (from (Platis et al. 1998)) proposed a general framework for carrying out performability analysis. Performability analysis provides a more detailed evaluation of the operational performance and is useful for performance evaluation of degrading systems (Platis et al. 1998). As pointed out by Aven (Aven 2008), performability analysis will be useful for 'highlighting the uncertainties beyond expected values and probabilities.'

In order to apply the Markov chain-based approach for performability analysis of bridge system, the condition assessment of adjacent girders need to be carried out. It may be difficult to acquire field data about the strain in prestressing strands of adjacent girders even using CSHM. Hence, a data-driven and reinforced learning model, namely, Polya urn model is proposed in the present study to assess the condition of the system. This model makes use of data of measured or monitored strains from limited number of girders located at random locations (i.e., condition of adjacency requirement has been relaxed) in a bridge system. The condition so assessed is used to evaluate the performance of PSC girder bridge system.

The methodology proposed in this paper is presented next.

4.5 Polya urn – based model for performance assessment

Let the reward structure at time t if given by $r_{CS}(t)$ (i.e., the PSC girder bridge system is assigned a reward rate $r_{CS_i}(t)$ if it is in condition state CSi at time t). Thus, the reward structure is a set of reward rates. Let he condition state of the system at time t is given by the vector CS(t) = [CS1(t), CS2(t), CS3(t), CS4(t), CS5(t)], and, let p(t) = [p1(t), p2(t), p3(t), p4(t), p5(t)] and r(t) = [rCS1(t), rCS2(t), rCS3(t), rCS4(t), rCS5(t)] be the corresponding probabilities and reward rates, respectively. Then, the instantaneous reward of the system at time t is (Bolch et al. 1998):

$$Z(t) = r_{CS(t)} \tag{6}$$

The expected instantaneous reward can be computed as,

$$E[Z(t)] = \sum_{i=1}^{5} r_{CS_i}(t) p_i(t) \tag{7}$$

The expected value computed using Eq. (7) defines the performability of the bridge system.

4.5.1 Specification of reward rates – proposed structure

The reward structure is the set of reward rates associated with the different system states. The reward structure relates possible system behaviour to a specified performance variable (refer Bolch et al. (Bolch et al. 1998) for some of the commonly used reward structures). The reward rates can be constant with time (stationary) or changing with time (dynamic). Since the PSC girders are normally designed for a long service life (typically 75 years), use of a dynamic reward rate is more rational. For a bridge system, it is more logical to assign a reward rate based on its age, design life, the time span it spends in a given condition state, and the variations in the demand and the operating environment. The determination of reward rates requires inputs from the designers, experts involved in inspection and condition assessment, and decision makers, and is

Table 1 Reward structure considered

Condition State of the Bridge System	% of girders with more than allowable prestress loss	Reward rate[a]
Very Good	0–20	10
Satisfactory	> 20–40	5
Poor	> 40–60	2
Serious	> 60–80	1
Critical	> 80–100	0.1

[a] The reward rate assumed in this paper is akin to the condition rating of the components of a bridge prescribed in United States National Bridge Inventory. An excellent review of the bridge condition rating is given in Omar and Nehdi (Omar and Nehdi 2018)

not within the scope of this study. Hence, in the present study, a stationary reward structure as given in Table 1 is assumed. This is one of the limitations of the reward structure considered. However, it does not affect the general methodology presented.

The step-by-step procedure for Polya Urn model-based performance assessment of given prestressed concrete bridge system with respect to prestress loss in tendons is as follows.

i). Let N and n be the total number and the number of monitored girders, respectively, in the PSC bridge system. The monitoring can be through use of concrete embedded strain gauges at the level of the centre of gravity of the prestressing steel (viz., vibrating wire strain gauges).
ii). Using the detailed procedure of AASHTO LRFD 2012, compute the expected values of loss of prestress at different times 't'. Since these prestress loss values are considered in deciding the initial prestress, these values are taken as the allowable prestress losses $P_{L\_all}(t)$ at the respective times.
iii). Using monitored strain data, estimate the actual prestress loss $(\mathrm{P}^{\mathrm{i}}_{\mathrm{L\_act}}(\mathrm{t}), \mathrm{i} = 1, 2, \ldots, \mathrm{n})$ at time t in the monitored PSC girders using Eq. 1.
iv). Determine the initial urn configuration (B, W), so that $B + W = n$, where B and W are the number of monitored PSC girders with $\mathrm{P}^{\mathrm{i}}_{\mathrm{L\_act}}(\mathrm{t}) < \mathrm{P}^{\mathrm{i}}_{\mathrm{L\_all}}(\mathrm{t}) \left(\mathrm{i} = 1, 2, \ldots, \mathrm{n}\right)$, and with $\mathrm{P}^{\mathrm{i}}_{\mathrm{L\_act}}(\mathrm{t}) \geq \mathrm{P}^{\mathrm{i}}_{\mathrm{L\_all}}(\mathrm{t})\ (\mathrm{i} = 1, 2, \ldots, \mathrm{n})$, respectively.
v). Compute the probability distribution of percentage of PSC girders with loss of prestress more than allowable at time t using Polya urn model (Eq. 5).
vi). Compute the probabilities (p_i, $i = 1, \ldots, 5$) for the bridge system being in different condition states (namely, Very Good, Satisfactory, Poor, Serious and Critical).
vii). Compute the bridge system performability using Eq. 7.

The schematic representation of the procedure for performance assessment of bridge system is shown in Fig. 1.

5 Illustrative example

This example demonstrates the usefulness of the methodology, presented in the previous section. A bridge system with one hundred nominally similar PSC bridge girders of span = 17.22 m and having dimensions as given by Garber et al. (Garber et al. 2016),

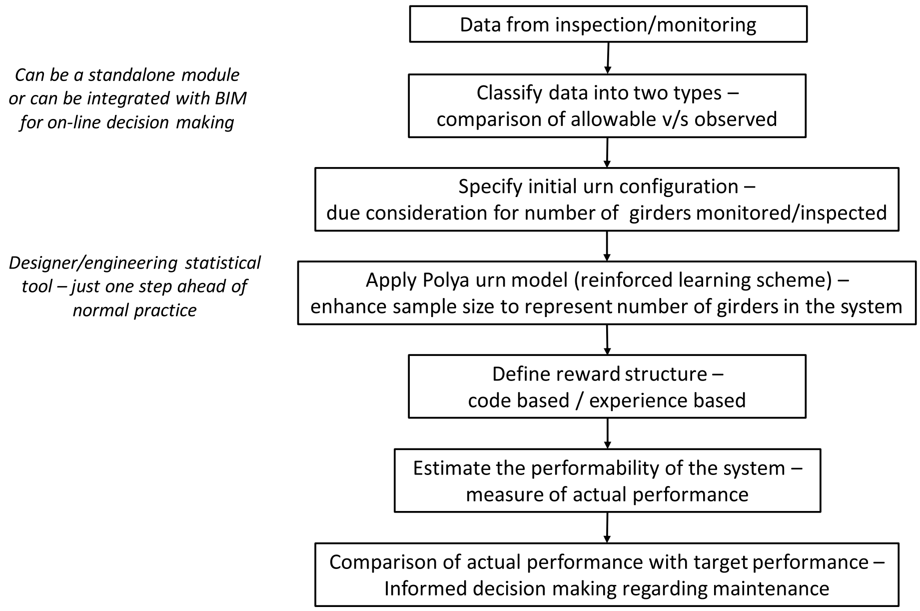

Fig. 1 Schematic representation of the Poly urn model-based procedure for performance assessment of bridge system

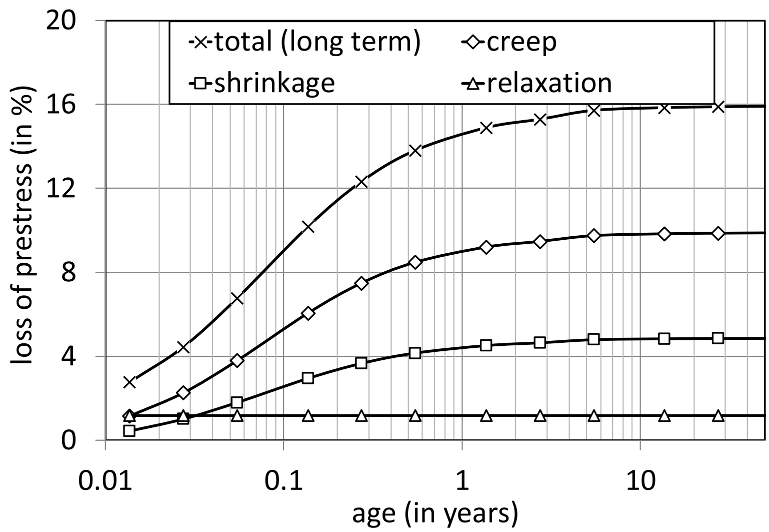

Fig. 2 Variation of prestress loss with age for PSC girder (using AASHTO LRFD 2012)

is considered (i.e., $N = 100$). The girders are of Type-C (1016 mm deep I section) having cross-sectional area $= 3.2 \times 10^5$ mm^2, area of prestressing steel $= 3748.38$ mm^2, stress at the time of transfer in prestressing steel $= 1398.9$ MPa, modulus of elasticity of prestressing steel $= 2 \times 10^5$ MPa, $f_{pu} = 1860.3$ MPa, moist curing duration $= 2$ days, relative humidity $= 60\%$, and age at transfer of prestress $= 1$ day. The exposure condition for all the one hundred girders is assumed to be nominally similar. It is assumed that 20 PSC girders (i.e., $n = 20$) at random are inspected/monitored after 10 years for strain.

6 Results and discussion

The AASHTO LRFD 2012 (AASHTO 2012) detailed method is used for estimating the loss of prestress due to shrinkage, creep, and relaxation. The estimated prestress losess at different times are shown in Fig. 2. From Fig. 2, it is noted that prestress loss due to creep is higher than those due to shrinkage and relaxation. The estimated values of total loss of prestress at different times are taken as the allowable ($P_{L\_all}(t)$), since these values would be used in the design.

It is assumed that at an age of 10 years from the transfer of prestress, the condition assessment of the bridge is being attempted. A total of 20 girders selected at random have been monitored for the strain (and hence stress) in tendons. The strains so obtained represent the observed strains. The number of girders with observed loss of prestress less than the estimated loss of prestress, n_1, and, those with observed loss of prestress more than the estimated loss of prestress, n_2, define the initial Polya Urn configuration for the bridge system considered. It is noted that $n_2 = n - n_1$. For five different initial configurations ((n_1, n_2) with varying values of n_1 and n_2), the probability distributions of percentage of girders in the PSC bridge system with loss of prestress more than the allowable are obtained using Eq. 5, typically for age of 10 years after transfer of prestress and are shown in Fig. 3. From Fig. 3, it is noted that the initial configuration has an influence on the skewness (measure of the asymmetry about the mean) of the probability distribution. When the initial configuration is such that n_1 and n_2 are closer to each other (for instance, the initial configuration (10, 10)), one obtains more or less symmetrical probability distributions. This is because the beta distribution with parameters n_1 and n_2 is the limiting probability distribution for the frequency of girders with loss of prestress more than the allowable, and for $n_1 = n_2$, the beta distribution is a symmetrical distribution (Mahmoud 2000).

Figure 4 shows the influence of initial configuration on the condition state probabilities of the bridge system. The different initial configurations considered represent some of the possible outcomes of the condition monitoring of randomly selected

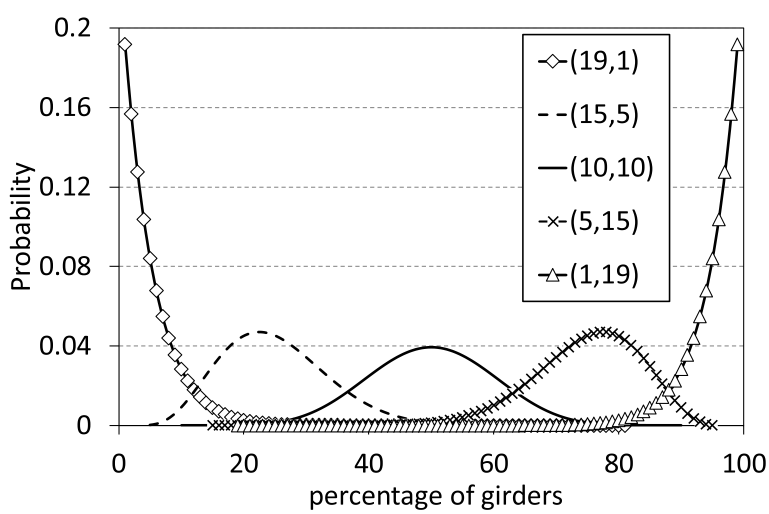

Fig. 3 Probability distributions for percentage of girders with loss of prestress more than allowable (time after transfer of prestress = 10 years, the values given in legend are the initial configurations considered)

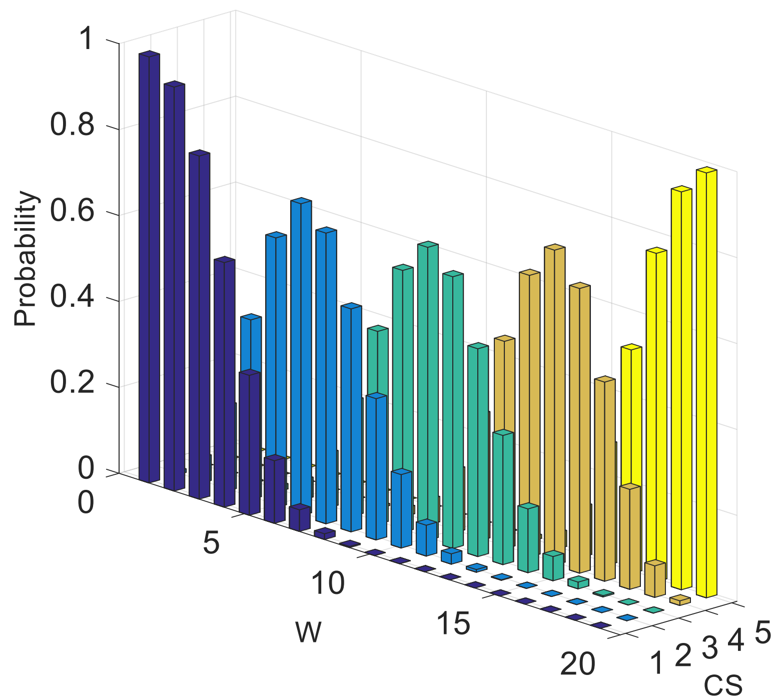

Fig. 4 Variation in condition state probabilities of the bridge system with variation in the initial configuration

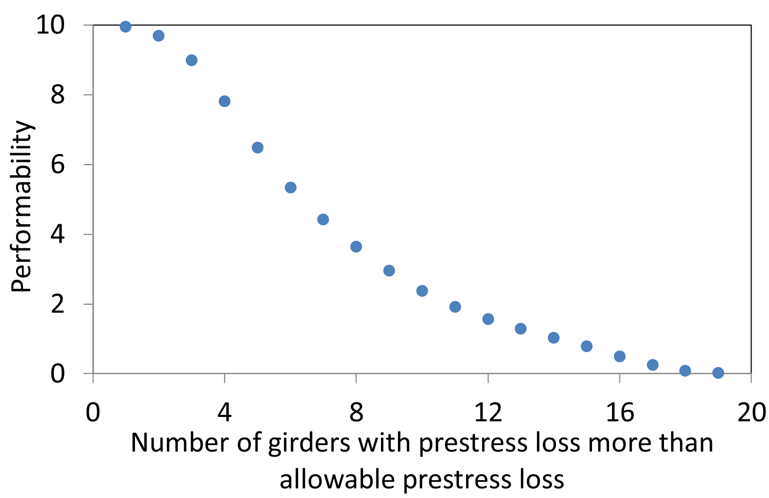

Fig. 5 Variation in performability with increase in number of girders having loss of prestress more than allowable in the initial configuration ($n=20$)

girders. It is noted from Fig. 4 that, as expected, the condition state of the bridge system changes from Very Good to Critical with increase in n_1. Using the condition state probabilities (Fig. 4) and the reward structure given in Table 1, the performability of the bridge system is determined (using Eq. 7), and is shown in Fig. 5. From Fig. 5, it is noted that as n_1 increases, the performability of the bridge system decreases, indicating the need for detailed inspection and undertaking remedial measures.

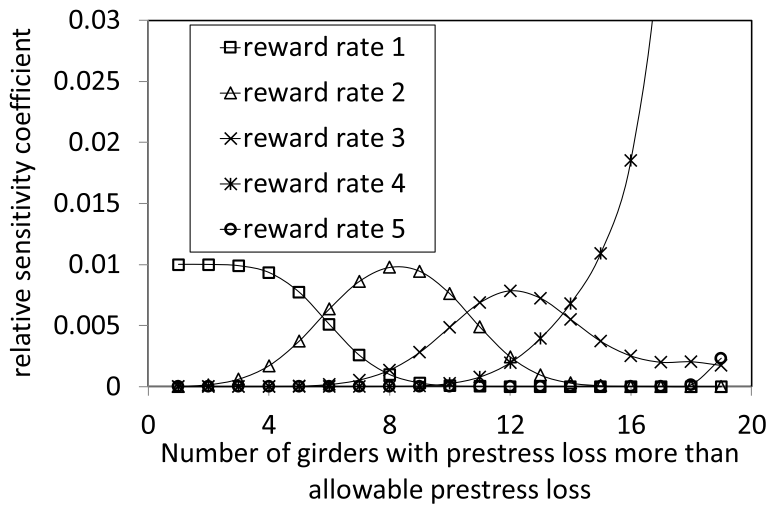

Fig. 6 Variation in relative sensitivity of performability with reward rate for different initial configurations ($n = 20$)

In order to study the relative importance of the variability of the reward rates considered and their influence on the performability, a sensitivity analysis is carried out. The relative sensitivity coefficients (defined as the fractional sensitivity of performability with respect to a fractional change in reward rate, i.e., $\frac{\partial(E[Z(t)])}{\partial(r_i(t))}\frac{r_i(t)}{E[Z(t)]}$) obtained are shown in Fig. 6. From this figure, it is noted that, as expected, the variation in relative sensitivity coefficients is similar to that of the condition state probabilities (see Fig. 4). However, when the probabilities of bridge system being in Serious and Critical condition states are high, the performability is more influenced by the reward rate for the Serious condition state.

Estimation of the performability of the bridge system at the different time instants will be useful in decision making regarding detailed inspection. For instance, suppose it is proposed to have a detailed inspection if 40% or more girders have loss of prestress more than the allowable. This indicates that the detailed inspection needs to be taken up when the performability of the bridge system falls below 5.0 (i.e., less than condition state Satisfactory, see Table 1). Thus, the proposed methodology provides a rational basis for taking informed decisions regarding inspection scheduling of the entire bridge system based on data from strain monitoring of a few number of girders.

7 Summary

A Polya urn model-based methodology for assessment of performance of prestressed concrete bridge system using data from strain monitoring in limited number of girders is presented in this paper. It is assumed that: (i) the environmental exposure condition is nominally similar for all of the PSC girders in the bridge system, and, (ii) the number of monitored girders are much less than the number of girders in the bridge system. The methodology can be used even when randomly selected girders in the bridge system are considered for monitoring. An example of a PSC bridge having one hundred bridge girders, with 20 PSC girders instrumented for strain monitoring is considered for illustrating the usefulness of the methodology. The performability of the system is computed considering estimated prestress loss from the monitored PSC girders. Estimating the performability of the system will be useful for decision making regarding scheduling of the detailed inspection of the bridge system.

7.1 Scope for future research

- In this paper only loss of prestress is considered. However, the actual condition of the girder that might have resulted in the loss of prestress (viz. cracking, some local distress) is not considered. This information will be available during preliminary/detailed inspections. Consideration of this information in performance assessment of bridge system using Polya urn model is going to be a challenging task.
- The reward rates associated with the condition states of the bridge system are assumed to be time-invariant in the present study, which can be further improved by consideration of time varying reward rates.

Appendix 1

As pointed out in the introduction, for bridge systems, one of the popular models used for condition assessment is Markov chain models. These models are highly mathematically tractable and are successfully used for, in general, off-line condition assessment of bridge stock (viz. Jiang and Sinha (Jiang and Sinha 1989)).

Depending on the understanding of the response variable characterizing the degradation of the performance of the system/girder and the amount of condition monitoring data available, the evolution of the response of the system, required for prediction of the condition state of the system in future or for scheduling of inspection, is modelled as continuous time- or discrete time- Markov chain.

If continuous time Markov chain is used for modeling the evolution of the response, determination of generator or rate matrix plays an important role. In general, the rate matrix is formulated using a detailed analytical modelling of the structure and the possible degradation mechanisms affecting the response evolution. The response of the girder defines the state space. Since the response evolves continuously in time the state space will be continuous. However, in order to take decisions, it is common to discretize the state space in to mutually exclusive and collectively exhaustive discrete states (Balaji Rao 2021). In this case each state corresponds to a range of response. One of the ways of discretisation is based on expected target reliability requirements at different times (Balaji Rao 2020). In such a case, the basic governing differential equation, modelling the response evolution of the system, is given by:

$$\frac{dP(t)}{dt} = \Lambda P(t) \tag{A-1}$$

where P(t) is the transition probability matrix at time t, Λ is the rate matrix. The steady state vector, π, corresponding to P(t) can be estimated from the following equations.

$$\left.\begin{array}{c}\pi P = \pi \\ \sum_{i=1}^{n} \pi_i = 1\end{array}\right\} \quad \text{(A-2)}$$

where n is the number of states of the Markov chain.

If the system response evolution is modelled using discrete time Markov chain, the formulations presented in Balaji Rao and Appa Rao (Balaji Rao and Appa Rao 1999; Balaji Rao and Appa Rao 2004) and Prakash Desayi and Balaji Rao (Desayi and Balaji 1989) can be used. The formulations presented above are amenable for updating based on information obtained from field observations (Balaji Rao and Appa Rao 2004).

Appendix 2

In this Appendix an attempt has been made to compare the estimated prestress loss, obtained using the procedure presented in Section 3.1, with the relevant experimental values. For this purpose, 16 nominally similar girders exposed to nominally similar environmental conditions are obtained from the database of girders created by Garber et al. (Garber et al. 2016). This exercise is taken up to examine whether the AASHTO LRFD 2012 (AASHTO 2012) method provides satisfactory results with respect to prestress loss estimation (required for applying the proposed method, Section 3). It is noted that results presented in this Appendix are from (Balaji Rao and Anoop 2019). However, these results are presented in this paper for the sake of completeness and for the ease of application.

The details of the sixteen girders, with dimensions and strength properties typically used in practice, are presented in Appendix 2: Table 2. In the last column the ratio of experimental to computed prestress loss are given for each girder. Neglecting the ratios equal to or greater than 2.0, the mean and coefficient of variation (cov) of the ratio are 1.29 and 0.23, respectively; these values may be considered satisfactory. Thus, the AASHTO LRFD2012 method can be used to compute the allowable prestress loss in the girders. It is noted that the computed statistics of the ratio compare satisfactorily with the mean and cov values (1.25 and 0.24, respectively) reported by Garber et al. (Garber et al. 2016) based on database of 237 girders.

Table 2 Details of the PSC bridge girders considered (Garber et al. (Garber et al. 2016))

Beam ID	Cement content (kg/m^3)	Fly ash content (kg/m^3)	Coarse aggregate (kg/m^3)	fine aggregate (kg/m^3)	water-cement ratio	$f_{c}{}^{r}{}_{i}$ (MPa)	Relative humidity (%)	Age at which prestress loss is measured (days)	Ratio of experimental-to computed-prestress loss (RATIO)
I-1	320.4	100.9	1097.6	723.8	0.34	48.3	49	980	1.166
I-2							65	939	1.015
I-3							65	948	1.082
I-4							65	962	1.213
I-5							49	975	1.052
I-6							49	973	0.958
I-7							65	946	1.015
I-8							65	966	0.995

Beam ID	Cement content (kg/m^3)	Fly ash content (kg/m^3)	Coarse aggregate (kg/m^3)	fine aggregate (kg/m^3)	water-cement ratio	$f'_{c\,i}$ (MPa)	Relative humidity (%)	Age at which prestress loss is measured (days)	Ratio of experimental-to computed-prestress loss (RATIO)
II-1	314.4	100.9	1168.7	777.2	0.22	45.5	49	955	2.371[b]
II-2							65	922	1.779
II-3							65	932	1.353
II-4							65	936	1.552
II-5							49	952	1.649
II-6							49	949	1.581
II-7							65	937	2.199[b]
II-8							65	923	1.598

[a] Computed using the procedure presented in Sect 3.1

[b] RATIOS equal to or greater than 2.0 are neglected in computing the statistics of RATIO

Abbreviations

AASHTO American Association of State Highway and Transportation Officials
CS Condition State
CSHM Continuous Structural Health Monitoring
LRFD Load and Resistance Factor Design
MC Markov Chain
NCHRP National Cooperative Highway Research Program
PSC Prestressed Concrete
RDSO Research Designs & Standards Organisation
RSHM Remote Structural Health Monitoring
SHM Structural Health Monitoring

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Authors' contributions

KBR: Conceptualisation of the study, Formulation of Polya Urn Model, Writing major portion of the paper, interpretation of results and in finalisation of the paper. MBA: Formulations related to performability, Developing computer codes and related numerical work, Writing some portions of the paper and finalisation of the paper. Both the authors read and approved the final manuscript.

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Availability of data and materials

In this paper the simulation based data are used. The computer codes and the inputs used for the purpose can be obtained by sending an email to: balaji@serc.res.in.

Declarations

Competing interests

No competing interests foreseen.

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References

AASHTO (2012) AASHTO LRFD Bridge design specifications. American Association of State Highway and Transportation Officials, Washington, DC

Abdel-Jaber H, Glisic B (2018) Monitoring of long-term prestress losses in prestressed concrete structures using fiber optic sensors. Struct Health Monit 18(1):254–269

ACI 423: Guide to Estimating Prestress Loss, ACI 423.10R-16, Joint ACI-ASCE Committee 423, American Concrete Institute, MI 48331 (2016)

Agrawal AK, Kawaguchi A (2009) Bridge element deterioration rates. Prepared for the New York state DOT, Report no. C-01-51. In the city college of New York. Dept. of Transportation, New York
Amin J, Adey B (2015) The effect of management decision processes on the management of bridges. J Civil Eng Manag 22(1):92–104
Antal T, Ben-Naim E, Krapivsky PV (2010) First-passage properties of the Polya urn process. J Stat Mechanics 2010(07):P07009
Aven T (2008) Risk analysis: assessing uncertainties beyond expected values and probabilities. John Wiley & Sons, West Sussex
Balaji Rao K (2020) Non homogeneous markov chain modelling of capacity degradation of RC beams against corrosion of reinforcement (Internal Report)
Balaji Rao K (2021) In: Singh SB, Madappa VR, Sivasubramanian HC (eds) Markov chain modelling of evolution of deflection in ferrocement flexural members, emerging trends of advanced composite materials in structural applications. Springer Nature Singapore Pvt. Ltd, Singapore, pp 67–96
Balaji Rao K, Anoop MB (2019) Polya urn model for assessment of pre-stress loss in pre-stressed concrete girders in a bridge system using limited monitoring data. In: Varde PV, Prakash RV, Joshi NS (eds) Risk Based Technologies. Springer Nature Singapore Pte Ltd, Singapore, pp 257–278
Balaji Rao K, Appa Rao TVSR (1999) A methodology for condition assessment of RC girder with limited inspection data. Bridge Struct Eng 29(4):13–26
Balaji Rao K, Appa Rao TVSR (2004) Stochastic modelling of crackwidth in reinforced concrete beams subjected to fatigue loading. Eng Struct 26(5):665–673
Barr P, Halling M, Boone S, Toca R, Angomas F (2009) UDOT's calibration of AASHTO's new Prestress loss design equations. Utah State University, Logan
Bazant ZP, Hubler MH, Yu Q (2011) Pervasiveness of excessive segmental bridge deflections: wake-up call for creep. ACI Struct J 108(6):766–774
Beaudry M (1978) Performance related reliability for computer systems. IEEE Trans Comput 27:540–547
Bolch G, Greiner S, de Meer H, Trivedi KS (1998) Queuing networks and Markov chains: modeling and performance evaluation with computer science applications. John Wiley & Sons Inc., New York
Bousquet O, Boucheron S, Lugosi G (2004) Introduction to statistical learning theory. Lect Notes Artif Intell 3176:169–207
Cousins TE (2005) Investigation of long-term prestress losses in pretensioned high performance concrete girders. Virginia Transportation Research Council, Charlottesville
Dai L, Bian H, Wang L, Potier-Ferry M, Zhang J (2020) Prestress loss diagnostics in pre-tensioned concrete structures with corrosive cracking. J Struct Eng ASCE 146:04020013–04020011
Desayi P, Balaji RK (1989) Markov chain model for cracking behaviour of reinforced concrete beams. J Struct Eng ASCE 115:2129–2144
Estes AC, Frangopol DM (2001) Bridge lifetime system reliability under multiple limit states. J Bridg Eng 6(6):523–528
Figueiredo LP, Santos M, Roisenberg M, Neto G, Figueiredo W (2014) Bayesian framework to wavelet estimation and linearized acoustic inversion. Geosci Remote Sensing Lett 11(12):2130–2134
Garber DB (2014) Effect of New Prestress Loss Estimation Procedure on Precast, Pretensioned Bridge Girders, Ph. D thesis, The University of Texas at Austin, May 2014, Austin, p 368
Garber DB, Gallardo JM, Deschenes DJ, Bayrak O (2016) Prestress loss database for pretensioned concrete members. ACI Struct J 113(2):313–324
Hallberg D (2005) Development and adaptation of a life-cycle management system for constructed work (licentiate thesis). KTH, Civil and Architectural Engineering, Gavle
Hearn G, Xi Y (2007) Service life and cost comparisons for four types of CDOT bridge decks. Prepared for the Colorado DOT, rep.No. CDOT-2007-2. University of Colorado, Boulder
Hughes AJ, Barthorpe RJ, Dervils N, Farrar CR, Worden K (2021) A probabilistic risk-based decision framework for structural health monitoring. Mech Syst Signal Process 150:107339
Jiang Y, Sinha KC (1989) Bridge service life prediction model using Markov chain. Transp Res Rec 1223:24–30
Kamatchi P, Balaji Rao K, Dhayalini B, Saibabu S, Parivallal S, Ravisankar K, Nagesh R (2014) Iyer: long-term prestress loss and camber of box girder bridge. ACI Struct J 111(6):1297–1306
Kesavan K, Ravisankar K, Parivallal S, Sreeshylam P (2005) Technique to assess the residual prestress in prestressed concrete members. Exp Tech 29(5):33–38
Klatter HE, Van Noortwijk JM (2003) Life-cycle cost approach to bridge management in the Netherlands. In: Proceedings of the 9th International Bridge Management Conference. Transportation Research Board, Orlando
Lavrenz S, Murillo-Hoyos J, Volovski MJ, Saeed TU, Labi S (2015) Service life and condition rating prediction of steel bridges. In: Proceedings of 8th international symposium on steel bridges: Innovation & new Challenges 2015 (SBIC-2015). Turkish Constructional Steelwork Association (TUCSA), Istanbul, pp 667–674
Liu A, Wang L, Bornn L, Farrar C (2019) Robust structural health monitoring under environmental and operational uncertainty with switching state-space autoregressive models. Struct Health Monit 18(2):435–453
Mahmoud HM (2000) Polya Urn Models. CRC Press, Boca Roston
Melhem HG, Cheng Y (2003) Prediction of remaining service life of bridge decks using machine learning. J Comput Civ Eng 17(1):1–9
Moomen M, Saeed TU, Ahmed A, Fang C, Labi S (2016) Empirical analysis of concrete bridge superstructure condition: influential factors and deterioration prediction. In: Transportation Research Board 95th Annual Meeting. Transportation Research Board, Washington, DC
Morcous G (2006) Performance prediction of bridge deck systems using Markov chains. J Perform Constr Facil 20(2):146–155
Morcrette, B.: 2012. Fully analyzing an algebraic Polya Urn model, arXiv:1203.4917v1 [math.CO] 22 Mar 2012
Narasinghe SB, Karunananda PA, Dissanayake PB (2006) Service life prediction of masonry arch bridges using artificial neural networks. In: Proceedings of the Concrete Bridge Conference: HPC: Build Fast, Build to Last. National Concrete Bridge Council, Reno

NCHRP (2012) Report 173, Volume 2: estimating life expectancies of highway assets. National Academy of Sciences, Washington DC
Ng S-K, Moses F (1996) Prediction of bridge service life using time-dependent reliability analysis. In: Harding JE, Parke GAR, Ryall MJ (eds) Bridge management 3: Inspection, maintenance, assessment and repair', Proceedings of Third International Conference on Bridge Management, University of Surrey. Taylor & Francis, Guildford, 14–17 April 1996
Nicolai RP (2008) Maintenance models for systems subject to measurable deterioration (Ph.D. thesis). Erasmus Universiteit Rotterdam, Netherlands
Omar T, Nehdi ML (2018) Condition assessment of reinforced concrete bridges: current practice and research challenges. Infrastructures 3(36):1–23
Pasanisi A (2014) Uncertainty analysis and decision-aid: methodological, technical and managerial contributions to engineering and R&D studies. Applications [stat. AP]. Université de Technologie de Compiègne, Compiègne, p 209
Platis A (2006) A generalized formulation for the performability indicator. Comput Math Appl 51:239–246
Platis A, Limnios N, Du ML (1998) Dependability analysis of systems modeled by non-homogeneous Markov chains. Reliab Eng Syst Saf 61:235–249
Ravisankar K, Sreeshylam P, Parivallal S, Kesavan K, Sridhar S, Sridharan N (2008) Final Project Report on Health Monitoring of a Flyover Bridge at Visakhapatnam Port Trust for the Extended Period (January 2006 to January 2008), Sponsored Project Report No. EML-SSP 05741–6, CSIR-SERC, Feb. 2008, p 35
Robelin CA, Madanat SM (2007) History-dependent bridge deck maintenance and replacement optimization with Markov decision processes. J Infrastruct Syst 13(3):195–201
Roller JJ, Russell HG, Bruce RN, Alaywan WR (2011) Evaluation of prestress losses in high-strength concrete bulb-tee girders for the Rigolets pass bridge. PCI J 56(1):110–134
Saeed TU, Moomen M, Ahmed A, Murillo-Hoyos J, Volovski M, Labi S (2017a) Performance evaluation and life prediction of highway concrete bridge superstructure across design types. J Perform Constr Facil 31(5):04017052. https://doi.org/10.1061/(ASCE)CF.1943-5509.0001051
Saeed TU, Qiao Y, Chen S, Alqadhi S, Zhang Z, Labi S, Sinha KC (2017b) Effects of bridge surface and pavement maintenance activities on asset rating (joint transportation research program publication no. FHWA/IN/JTRP-2017/19). Purdue University, West Lafayette
Saeed TU, Qiao Y, Chen S, Gkritza K, Labi S (2017c) Methodology for probabilistic modeling of highway bridge infrastructure condition: accounting for improvement action effectiveness and incorporating random effects. J Infrastruct Syst 23(4):04017030
Saeed TU, Qiao Y, Moomen M, Ahmed A, Labi S (2016) Discrete-outcome modeling of bridge component deterioration: accounting for the effects of maintenance and observation-specific correlation using random effects. Maintenance, monitoring, safety, risk and resilience of bridges and bridge networks. In: 8th International Conference on Bridge Maintenance, Safety and Management, Brazil, CRC Press, London, p 380
Saliya MM (2008) Case study on launching of PSC box girders in mass rapid transit system (MRTS) elevated structures, Project Report. Indian Railway Institute of Civil Engineering, Pune
Sánchez-Silva M, Frangopol DM, Jamie Padgett J, Soliman M (2016) Maintenance and operation of infrastructure systems: review. J Struct Eng 142(9). https://doi.org/10.1061/(ASCE)ST.1943-541X.0001543
Sanders WH, Meyer JF (1991) A unfied approach for specifying measures of performance, dependability ad performability. In: Avizicuis A, Lapric J (eds) Dependable Computing for Critical Applications. Springer-Verlag, Vienna, pp 515–237
Schöbi R, Chatzi EN (2016) Maintenance planning using continuous-state partially observable Markov decision processes and non-linear action models. Struct Infrastruct Eng 12(8):977–994
Sen D, Nagarajaiah S (2018) Data-driven approach to structural health monitoring using statistical learning algorithms. In: Ottaviano E, Pelliccio A, Gattulli V (eds) Mechatronics for cultural heritage and civil engineering. Intelligent systems, control and automation: science and engineering, Vol 92. Springer, Cham
Smith RM, Trivedi KS, Ramesh AV (1988) Performability analysis: measures, an algorithm and a case study. IEEE Trans Comput 37(4):406–417
Sobanjo JO (2011) State transition probabilities in bridge deterioration based on Weibull sojourn times. Struct Infrastruct Eng 7(10):747–764
Swartz BD, Scanlon A, Schokker AJ (2012) AASHTO LRFD bridge design specifications provisions for loss of prestress. PCI J Fall 2012:108–132
Tolliver D, Pan L (2011) Analysis of bridge deterioration rates: a case study of the Northern Plains region. J Trans Res Forum 50(2):87–100
Van Noortwijk JM, Klatter HE (2004) The use of lifetime distributions in bridge maintenance and replacement modelling. Comput Struct 82(13–14):1091–1099
Veshosky D, Beidleman C, Buetow G, Demir M (1994) Comparative analysis of bridge superstructure deterioration. J Struct Eng 120(7):2123–2136
Worden K, Manson G (2006) The application of machine learning to structural health monitoring. Proc R Soc A 365:515–537
Yang Y, Myers JJ (2005) Prestress loss measurements in Missouri's first fully instrumented high-performance concrete bridge. Trans Res Record 1928(1):118–125
Ye C, Butler L, Calka B, Iangurazov M, Lu Q, Gregory A, Girolami M, Middleton C (2019) A digital twin of bridges for structural health monitoring. In: 12th International Workshop on Structural Health Monitoring, September 2019
Zhang Z, Sun X, Wang X (2003) Determination of bridge deterioration matrices with state national bridge inventory data. In: Proceedings of the 9th international bridge management conference of the Transportation Research Board, Transportation Research Board, Washington, DC, pp 207–218

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