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Pedestrian-induced footbridge vibration response based on a simple beam analytical method

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Abstract

This article aims to study the quantitative calculation of pedestrian-induced footbridge vibration comfort. Firstly, the analytical expression for the vibration response was derived. In addition, the simplified formula of the vibration response under resonance condition was put forward. The proposed analytical solution was compared with the numerical solution and the experimental result. Secondly, the analytical method was used to calculate the acceleration response under different crowd densities. The peak acceleration distribution and cumulative probability were analyzed. Finally, the cumulative probability that exceeded the acceleration limit was proposed as the comfort evaluation index, and the improved annoyance rate model was used to verify the proposed evaluation method. The results demonstrate that the analytical method can efficiently calculate the dynamic response of footbridges. Errors between the analytical results and the experimental results are less than 6.2%. Vibration comfort is negatively correlated with crowd density and walking speed. Furthermore, the errors between the proposed discomfort probability values and the calculated annoyance rate results are within 6%.

Keywords: Pedestrian-induced vibration, Analytical solution, Vibration comfort, Cumulative probability

1 Introduction

Modern footbridges are gradually developing in the direction of low frequency and lightweight. Vibration fundamental frequency and pedestrian walking step frequency are in the same range, which can easily trigger vibration comfort problems. Hence, vibration comfort becomes a critical issue in the design of footbridges (Han et al. 2013). This paper mainly discusses the vibration response under crowd load excitation and the pedestrian-induced footbridge vibration comfort.

Establishing an accurate pedestrian-induced load model is one of the key issues in calculating the vibration response of footbridges (Cao 2016). The earliest single-person load model dates back to 1979, it assumed that the pedestrian step frequency was consistent with the footbridge fundamental frequency, and the model was widely used because of its simplicity BSI (British Standards Institution) (1979). In 2005, Blanco et al. (2005)

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proposed a deterministic single-person load model considering the sum of the third-order harmonic loads and suggested reasonable dynamic load coefficient and phase difference. Zicanovic and Pavic (2011) established a time-domain stochastic model from the perspectives of statistics and probability theory, and the analysis showed that considering the non-periodicity of the walking load was more consistent with the actual situation. Ding and Mi 2013) equivalently regarded the walking load as a multi-level harmonic load and analyzed the dynamic response of the structure under the excitation of three vibration sources by using the weighted power spectrum. Ning (2012) presented a simple and efficient pseudo-random time-domain model based on the Zivanovic model. Jian et al. (2010) put forward a more refined walking load model, and analyzed the relationship between step frequency and walking speed. Mullarney and Archbold (2013) used force plates for real measurements, and the experimental results showed a high correlation between dynamic load factor and pedestrian walking speed.

Research on pedestrian-induced footbridge vibration problems focuses on vibration response analysis Ramos et al. (2020). At present, the analytical method and numerical method are often used to calculate the pedestrian-induced vibration response of footbridges (Garinei 2006; Garinei and Risitano 2008). Tadeu et al. (2022) determined the structural form of the bridge under static load and the eigenfrequencies excited under dynamic load by numerical method. Chen and Liu (2009) explained the whole process of footbridge dynamics design by using ANSYS modeling. Chen et al. (2018) used the analytical method to derive the vibration response expression of a simply supported beam under a single-person load. Basaglia et al. (2021) compared the vibration response of large span slabs under different pedestrian loads based on numerical studies. The comparison results showed that the accuracy of the response calculation depended on the rationality of the load model.

Aiming at pedestrian-induced footbridge vibration comfort, the design specifications of footbridges in various countries have corresponding provisions for pedestrian comfort. Usually, the method of controlling the frequency and acceleration is used to ensure that the dynamic characteristics of the structure meet the requirements. AASHTO (2008) stipulates that the first-order vertical frequency of the footbridge should be greater than 3.0 Hz, ISO 2631-1 ISO (1997), HIVOSS (2008), and BSI (British Standards Institution) (1979) have provided different acceleration limits. Scholars have carried out extensive research on the evaluation of pedestrian-induced footbridge vibration comfort. Feng et al. (2013) performed actual dynamic tests on a large number of footbridges, and questionnaire surveys on pedestrian comfort were conducted. Based on the investigation's results, an index to quantitatively describe the actual comfort of pedestrians was established. Liu 2010 investigated the pedestrian-induced vibration comfort of a footbridge in Wuhan by combining finite element analysis and field measurement, and evaluated the vibration comfort conditions of the footbridge according to the British standard BS5400. Zhu et al. (2016) studied the vibration comfort of a single person crossing a bridge at different speeds and assessed it with probability values. Chen et al. 2021proposed a sensible model for pedestrian-induced vibration comfort calculation based on the vibration endurance capacity of pedestrians and the vibration response of footbridges.

The above studies provide valuable explorations from different perspectives for assessing pedestrian-induced footbridge vibration comfort. Nevertheless, there are still challenges in the following areas:

(1). The calculation of structural vibration response is a complex problem (Chen 2008; Yuan 2006). The analytical solution is an exact solution (Francesco et al. 2011). However, there are few studies that use analytical methods to solve the dynamic response of structures under multi- pedestrian excitation. Moreover, the analytical method is not easily applied due to the complexity of the solution process.

(2). In fact, footbridge vibration is mostly caused by crowd load excitation. Due to the randomness of pedestrian gait parameters, the vibration response of the structure presents a probability distribution in a certain area, so it is unreasonable to use a single working condition for comfort evaluation. In addition, the traditional comfort index cannot quantitatively evaluate the structural vibration comfort under multiple working conditions.

In view of the existing problems, this study deduces the analytical solution expression of the structural vibration response under crowd load excitation. Meanwhile, the peak accelerations under different crowd densities and walking speeds are calculated using the analytical method. The distributive law and cumulative probability of the peak acceleration are analyzed. Finally, the probability value that exceeds the acceleration peak is proposed as the quantitative index of comfort, which provides a reference for evaluating pedestrian-induced footbridge vibration comfort.

2 Structural response and simplified calculation under crowd load excitation

2.1 Analytical solution of structural response under crowd load excitation

Assuming that m expresses the mass per unit length of the footbridge, c expresses the footbridge damping, EI expresses the bending stiffness, and v is the velocity of a pedestrian. When the mass of pedestrian moving load is far less than that of beam, ignoring the inertia force of load mass can be simplified as the crowd load model shown in Fig. 1.

Based on the periodicity of walking force, the vertical excitation generated by the i-th pedestrian is expressed as the sum of the body weight and the harmonic load of order k, which is expressed as:

$$F_i(t) = G\left[1 + \sum\nolimits_{k=1}^{n} \alpha_{ki} \sin\left(2\pi k f_{pi} t - \varphi_{ki}\right)\right] \tag{1}$$

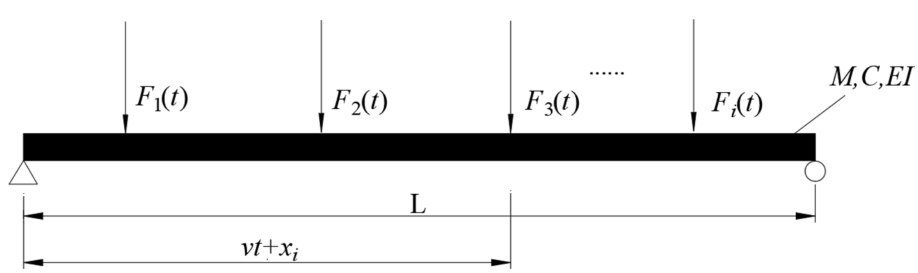


Fig. 1 Simple beam model under multi-pedestrian excitation

Where f_{pi} expresses the step frequency of the i-th pedestrian, G expresses the average weight of pedestrians, k expresses the order of simple harmonics, α_{ki} expresses the dynamic factor of the i-th pedestrian at the k-th order walking load component, ϕ_{ki} is the phase of the i-th pedestrian in the k-th simple harmonic, t is time. Compared with the generated response by the walking load component, the generated response by the pedestrian's dead weight can be ignored:

$$F_i(t) = G\sum_{k=1}^{n}\alpha_{\mathrm{ki}}\sin\left(2\pi kf_{pi}t - \varphi_{ki}\right) \tag{2}$$

The differential formula of pedestrian-induced footbridge vibration can be expressed as:

$$\left|EI(x)y''\right|'' + m(x)ij = \delta(x_i - v_it + d_i)F_i(t) \tag{3}$$

Where x_i expresses the position of the i-th pedestrian at time t, v_i expresses the walking speed of the i-th pedestrian, d_i is the initial position of the i-th pedestrian. δ is Dirac function. The relationship between pedestrian walking frequency and walking speed is as follows:

$$f_{pi} = 0.35{v_i}^3 - 1.59{v_i}^2 + 2.93v_i \tag{4}$$

From formula (4), we can obtain:

$$v_i = \sqrt[3]{-\frac{2-f_{pi}}{0.7} + \sqrt{\left(2-\frac{f_{pi}}{0.12}\right)^2}} + \sqrt[3]{-(2-f_{pi})/0.7 - \sqrt{(2-f_{pi})^2/0.12}} + 1.51 \tag{5}$$

The step length of the pedestrian is:

$$l_i = v_i/f_{pi} \tag{6}$$

Converting the damping term into a differential formula expressed in modal coordinates:

$$\ddot{q}(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2q_n(t) = \frac{P_n}{M_n} \tag{7}$$

The n-th order walking mode force under pedestrian load is expressed as:

$$P_n = \int_0^l \delta(x_i - v_it + d_i)F_i(t)\phi_n(x)dx = \phi(v_it + d_i)F_i(t) \tag{8}$$

The vibration formula of the crowd load at a certain time on the structure can be expressed as:

$$\ddot{q}(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2q_n(t) = \frac{1}{M}\sum_i^N\phi(v_it + d_i)F_i(t) \tag{9}$$

Where N is the total number of people acting on the structure, which can be calculated from the footbridge area and crowd density. The oscillation function can be approximated as a half-sine function:

$$\phi(v_it + d_i) = \sin\pi(v_it + d_i)/L \tag{10}$$

For the crowd load, only the first-order component of the load excitation generally needs to be considered, because its higher-order harmonics have less synchronous tunability than the fundamental harmonics. The vibration response of the footbridge can be accurately reflected by taking the first-order mode. The correlation between dynamic load factor and pedestrian walking speed was shown to be significant through experimental validation, α_1 values for:

$$\alpha_1 = 2.5\left(0.111v^2 - 0.017v\right) \tag{11}$$

By substituting formula (10), it can be rewritten as:

$$\ddot{q}(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{G\alpha_1}{M}\sum_i^N \sin\left(2\pi f_{pi}t - \varphi_i\right)\sin\frac{\pi(v_i t + d_i)}{L} \tag{12}$$

The above formula can be transformed by trigonometric transformation into:

$$\begin{aligned}\ddot{q}(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = &\frac{G\alpha_1}{mL}\sum_i^N \cos\frac{n\pi d_i}{L}\left[\cos\left(\omega_{1in}t\right) - \cos\left(\omega_{2in}t\right)\right] \\ &+ \frac{G\alpha_1}{mL}\sum_i^N \sin\frac{n\pi d_i}{L}\left[\sin\left(\omega_{1in}t\right) + \sin\left(\omega_{2in}t\right)\right]\end{aligned} \tag{13}$$

Where: $\omega_{1\text{in}} = 2\pi f_{pi}\left(1 - \frac{l_i}{2L}\right), \omega_{2\text{in}} = 2\pi f_{pi}\left(1 + \frac{l_i}{2L}\right)$. In order to simplify the calculation, the damping ratio of the footbridge $\zeta_n \cong 0$ was substituted into the above formula for solving, and the following formula was obtained:

$$\begin{aligned}\ddot{q}(t) = &\frac{\alpha_1 G}{mL\omega_1^2}\sum_i^N \cos\left(\frac{n\pi d_i}{L}\right)\begin{bmatrix} A_{1in}\omega_{1\text{in}}^2\cos(\omega_{1in}t + \phi_{1in}) + A_{2in}\omega_{2\text{in}}^2\cos(\omega_{2in}t + \phi_{2in}) + \\ S_{1in}A_{B1in}e^{-\zeta_n\omega_n t}\omega_{\text{B1in}}^2\cos(\omega_{B1in}t + \phi_{B1in}) \end{bmatrix} \\ &\frac{\alpha_1 G}{mL\omega_1^2}\sum_i^N \sin\left(\frac{n\pi d_i}{L}\right)\begin{bmatrix} A_{1in}\omega_{1\text{in}}^2 \sin(\omega_{1in}t + \phi_{1in}) + A_{2in}\omega_{2\text{in}}^2 \sin(\omega_{2in}t + \phi_{2in}) + \\ S_{1in}A_{B1in}e^{-\zeta_n\omega_n t}\omega_{\text{B1in}}^2 \sin(\omega_{B1in}t + \phi_{B1in}) \end{bmatrix}\end{aligned} \tag{14}$$

In formula (14):

$$\begin{aligned}A_{1in} &= \frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega_{1in}^2\right) + \left(2\zeta_n\omega_n\omega_{1in}\right)^2}}, \\ A_{2in} &= \frac{-\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega_{2in}^2\right) + \left(2\zeta_n\omega_n\omega_{2in}\right)^2}}\tan\phi_{1in} \\ &= \frac{\omega_n^2 - \omega_{1in}^2}{2\zeta_n\omega_n\omega_{1in}}, \tan\phi_{2in} = \frac{\omega_n^2 - \omega_{2in}^2}{2\zeta_n\omega_n\omega_{2in}}A_{B1in} \\ &= \sqrt{C_{1in}^2 + D_{1in}^2}\tan\phi_{B1in} = \frac{D_{1in}}{C_{1in}}, S_{1in} \\ &= -\operatorname{sgn}\left(C_{1in}\right)C_{1in} = \frac{\zeta_n}{\omega_n^2\sqrt{1 - {\zeta_n}^2}} \\ &\left[\left(\omega_n^2 - \omega_{1in}^2\right)A^2{}_{1in} - \left(\omega_n^2 - \omega_{2in}^2\right)A^2{}_{2in}\right]D_{1in} \\ &= \frac{1}{\omega_n^2}\left[\left(\omega_n^2 - \omega_{1in}^2\right)A^2{}_{1in} - \left(\omega_n^2 - \omega_{2in}^2\right)A^2{}_{2in}\right]\end{aligned}$$

2.2 Simplified calculation of footbridge resonance under crowd load excitation

The structural dynamic response method is usually used in the current specification to evaluate the comfort of footbridges. This method uses the generated maximum response on the footbridge under resonance to assess its vibration practicality. It ensures that the vibration response under walking load does not exceed the comfort threshold value. Therefore, we introduce the reasonable assumption that pedestrians are evenly distributed on the footbridge and walk on the footbridge at the same speed. In this way, the actual moving load can be equivalent to the fixed load of "walking in place". When the footbridge resonates, considering only the first-order harmonics of the walking force, the crowd walking force is expressed as:

$$P_n(x,t) = \sqrt{N} G\alpha sin\left(2\pi f_i t\right) \sum\nolimits_i^N \delta(x - x_i - vt) \tag{15}$$

The differential formula of motion when the footbridge resonates is:

$$\ddot{q}(t) + 2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{\sqrt{N}G\alpha_1}{mL}[\cos(\omega_{1n}t) - \cos(\omega_{2n}t)] \tag{16}$$

Where: $\omega_{1n} = \omega_n\left(1 - \frac{\zeta_n}{\varepsilon_n}\right), \omega_{2n} = \omega_n\left(1 + \frac{\zeta_n}{\varepsilon_n}\right), \varepsilon_n = \frac{p\zeta_n}{n}$. The solution of this formula is:

$$q_n(t) = \frac{\sqrt{N}\alpha_1 G}{mL\omega_n^2}\left[\begin{array}{c} A_{1n}\sin(\omega_{1n}t + \phi_{1n}) + A_{2n}\sin(\omega_{2n}t + \phi_{2n}) + \\ S_n A_{Bn} e^{-\zeta_n\omega_n t}\sin(\omega_{Bn}t + \phi_{Bn}) \end{array}\right] \tag{17}$$

In the above formulas:

$$A_{1n} = \frac{\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega_{1n}^2\right) + (2\zeta_n\omega_n\omega_{1n})^2}}, A_{2n} = \frac{-\omega_n^2}{\sqrt{\left(\omega_n^2 - \omega_{2n}^2\right) + (2\zeta_n\omega_n\omega_{2n})^2}}$$

$$\tan\phi_{1n} = \frac{\omega_n^2 - \omega_{1n}^2}{2\zeta_n\omega_n\omega_{1n}}, \tan\phi_{2n} = \frac{\omega_n^2 - \omega_{2n}^2}{2\zeta_n\omega_n\omega_{2n}}, A_{\mathrm{B}1n} = \sqrt{C_{1n}^2 + D_{1n}^2}$$

$$\tan\phi_{B1n} = \frac{D_{1n}}{C_{1n}}, S_{1in} = -\operatorname{sgn}(C_{1n}), C_{1n} = \frac{\zeta_n}{\omega_n^2\sqrt{1 - {\zeta_n}^2}}\left[\left(\omega_n^2 - \omega_{1n}^2\right)A^2{}_{1n} - \left(\omega_n^2 - \omega_{2n}^2\right)A^2{}_{2n}\right]$$

$$D_{1n} = \frac{1}{\omega_n^2}\left[\left(\omega_n^2 - \omega_{1n}^2\right)A^2{}_{1n} - \left(\omega_n^2 - \omega_{2n}^2\right)A^2{}_{2n}\right]$$

Since the damping ratio of the footbridge is generally: $0.2\% \leq \zeta_n \leq 3.0\%$, which can be simplified by substituting $n/p = \zeta_n/\varepsilon_n \to 0$ into the formula as:

$$\begin{aligned} A_{1n} &= \frac{\varepsilon_n}{2\zeta_n}\frac{1}{\sqrt{1 + {\varepsilon_n}^2}} = -A_{2n}, \tan\phi_{1n} = \frac{1}{\varepsilon_n} \cong -\tan\phi_{2n}, A_{B1n} \\ &= \sqrt{C_{1n}^2 + D_{1n}^2} S_{1in} = -\operatorname{sgn}\left(C_{1n}\right), C_{1n} = -\frac{\varepsilon_n}{1 + {\varepsilon_n}^2} D_{1n} \\ &= -\frac{\varepsilon_n}{\zeta_n\left(1 + {\varepsilon_n}^2\right)}, \tan\varphi_{B1n} \cong \frac{1}{\zeta_n} \to \infty \varphi_{B1n} \to \frac{2}{\pi}, s_n = -1, \omega_{Bn} \\ &= \omega_n\sqrt{1 - \zeta_n^2} \cong \omega_n \end{aligned}$$

Formula (17) can be simplified as:

Fig. 2 A footbridge in Jingzhou City

$$\ddot{q}(t) \cong -\frac{\sqrt{N}\alpha_1 G}{ml}\left\{\frac{\varepsilon_n}{\zeta_n}\frac{1}{1+\varepsilon_n^2}\left[\sqrt{1+\varepsilon_n^2}\cos\left(\frac{\zeta_n}{\varepsilon_n}\omega_n t\right)+e^{-\zeta_n\omega_n t}\right]\right\} \tag{18}$$

Formula (18) can be used to estimate the maximum vibration response of the footbridge at resonance.

2.3 Analysis of time history response

In order to verify the accuracy of the proposed analytical method, field experiments were conducted on a footbridge in Jingzhou City. The footbridge connects Wanda Street and Huafu Square with a single-span structure, as shown in Fig. 2. Combined with the finite element method, the proposed analysis method was compared with the finite element method and the measured value.

The finite element analysis software ANSYS is used to establish the model. According to the structural characteristics of footbridges, the beam and rod elements are used to simulate the whole structure when establishing the structural dynamic analysis and calculation mechanical model. The footbridge is divided into 70 elements, a total of 71 nodes, as shown in Fig. 3. The footbridge span $L = 15\,m$, the linear density $m = 1400\,\mathrm{kg/m}$, the bending stiffness $EI = 3 \times 10^9 \mathrm{N} \bullet \mathrm{m}^2$, and the simply supported girder fundamental frequency $f = 1.877\,\mathrm{Hz}$. The mean and standard deviation of walking frequency corresponding to different population densities are shown in Table 1.

In the field experiment, the vertical accelerometer is installed in the middle of the bridge span, and the INV3018 portable data acquisition instrument is used to collect the vibration acceleration signal, as shown in Figs. 4 and 5. Photos of the field experiments are shown in Fig. 6. In order to reduce the experimental error, several experiments were

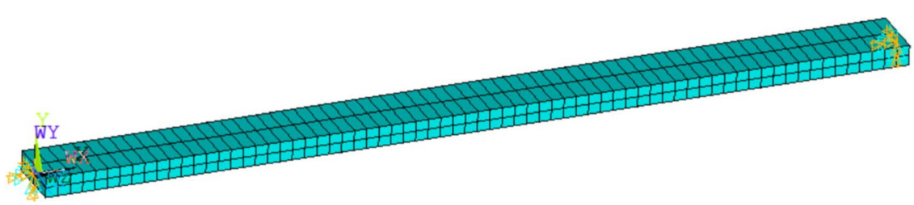


Fig. 3 The footbridge model

Table 1 Step frequency and weight corresponding to crowd density

Crowd density/P/m^2	Walking frequency/Hz		Weight/Kg
	Average value	Standard deviation	Average value
0.2	1.87	0.186	64.6
0.4	1.88	0.18	
0.6	1.856	0.176	

carried out under the same set of working conditions. Taking peak acceleration and root mean square acceleration as the comfort index, the measured average values are compared with the calculated values of the analytical method and finite element method.

Due to the large randomness of the crowd load, there is a big difference between the simulated pedestrian load and the actual pedestrian load, and the calculated time-history response is not comparable to the measured time-history response. Therefore, the time-history responses calculated by the analytical method and finite element method are compared in this paper. Formula (14) and finite element method are used to calculate the time-history response of the structure under different population densities, as shown in Fig. 7.

As can be seen from Fig. 7, the calculation results of formula (14) can better envelop the time history values of the finite element method, it demonstrates that the calculation method is reliable. Moreover, the simplified formula is more efficient under the premise of ensuring accuracy.

The acceleration response of the footbridge under different crowd densities is obtained by the three methods as shown in Table 2. From the analysis in Table 2, it is visible that the calculation results of the finite element method and experimental results are smaller than those of the analytical method. The main reason is that the proposed analytical method does not consider the damping effect of the footbridge. The errors between the calculation results of formula (14) and experimental results for the three working conditions are 4.8%, 6.2%, and 5.7% respectively. It can provide a reference for the vibration response calculation of similar footbridges.

Fig. 4 The vertical accelerometers

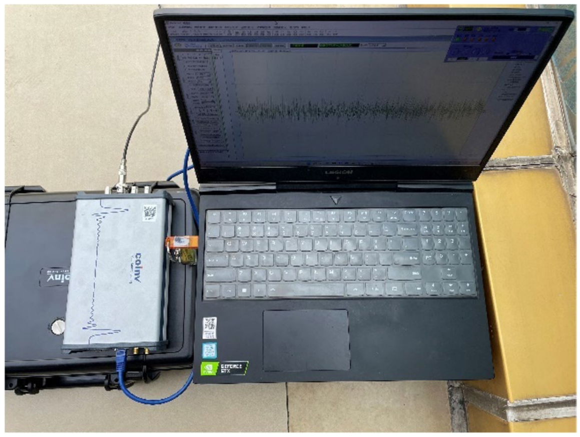


Fig. 5 The INV3018 data acquisition instrument and computer

3 Analysis of acceleration response under different crowd density

Since the impact of pedestrian step length on the acceleration response is small, the walking speed plays a key role in the distribution of response (Han et al. 2013). This paper further analyzes the effects of pedestrian gait parameters and crowd density on the vibration response of the footbridge.

According to the existing research (Chen et al. 2014), the average walking frequency of slow, medium, and fast walking is 1.86 Hz, 2.10 Hz and 2.35 Hz, and the standard deviation is 0.14 Hz, 0.15 Hz and 2.35 Hz. When the population density is 0.2, 0.4, and 0.6 P/m^2, the effect of pedestrian interaction is small. Therefore, the relationship between crowd density and pedestrian walking frequency can be considered independent, and pedestrian walking speed can be calculated by formula (5). Based on the above relationship between pedestrian walking frequency and walking speed, 1000 pedestrian loads were generated for each group of working conditions. According to formula (14), the structural dynamic response at different crowd density and walking speed combinations is obtained by programming calculation. Figure 8 shows that when the crowd density is 0.2 P/m^2, the probability of peak acceleration below 0.05 m/s^2 in slow-pass mode reaches 49.2%, while the probability of acceleration below 0.1 m/s^2 increases to 69.8%. Meanwhile, the maximum peak acceleration can reach up to 0.647 m/s^2, but its distribution is negligible. Unlike slow speed, the generated maximum peak acceleration at medium and

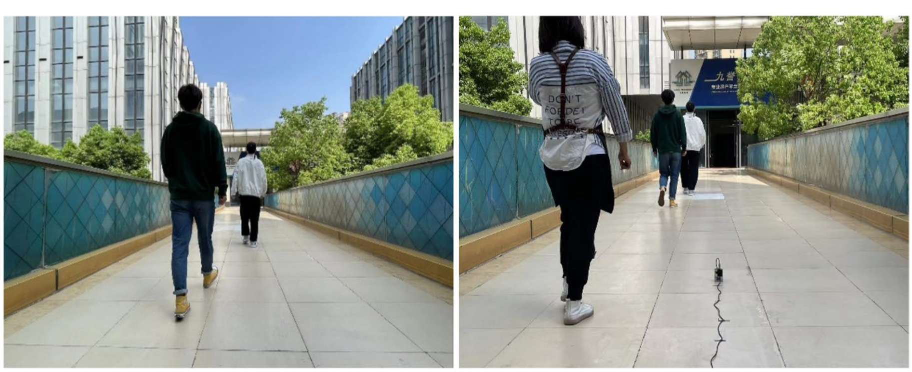


Fig. 6 Field experiment

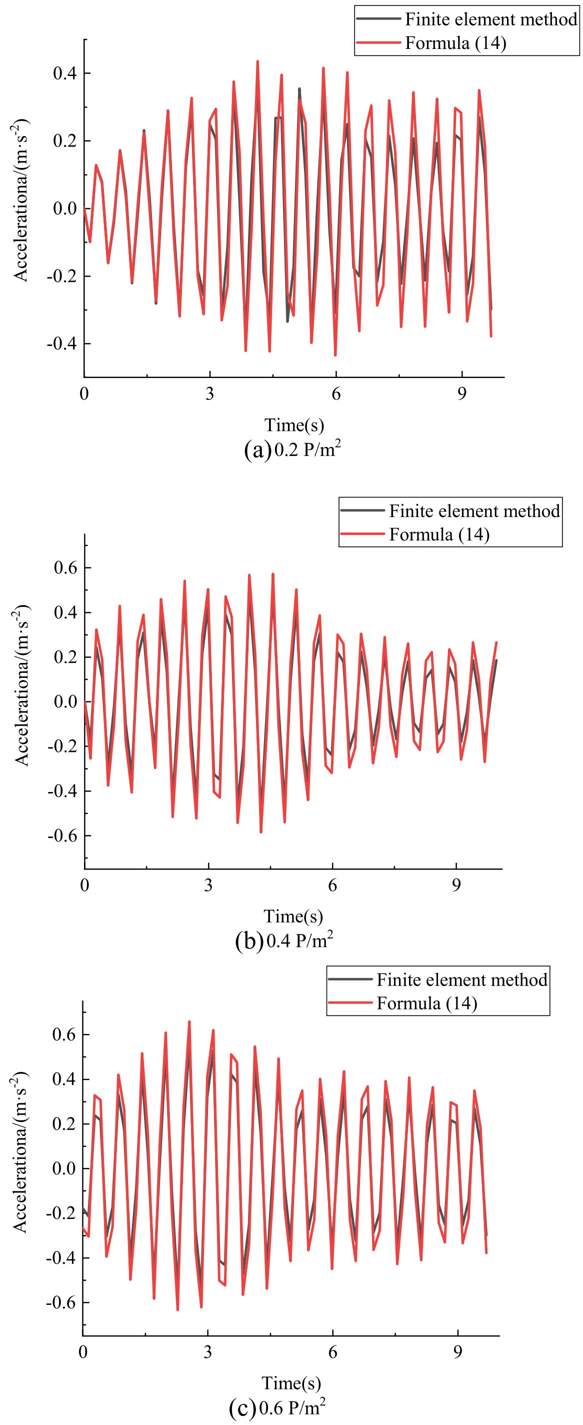


Fig. 7 Comparison of two acceleration time history curve under different crowd densities

high speeds through the footbridge is 0.784 m/s^2 and 0.973 m/s^2, while the probability of distribution corresponding to peak acceleration below 0.05 m/s^2 was reduced to 33.8% and 18.8%. The cumulative probability of acceleration less than 0.1 m/s^2 in two cases is 58.8% and 48.0%, respectively.

Table 2 Comparison between experimental results and two calculation results

Crowd density/ P/m^2	a_{max}/m·s^{-2}			a_{rms}/m·s^{-2}		
	experimental results	Formula (14)	numerical solution	experimental results	Formula (14)	numerical solution
0.2	0.415	0.436	0.377	0.229	0.241	0.206
0.4	0.527	0.562	0.512	0.281	0.314	0.268
0.6	0.603	0.639	0.596	0.364	0.377	0.345

Figures 9 and 10 show the probability distribution of structural vibration response under the crowd density of 0.4 P/m^2 and 0.6 P/m^2. It can be seen from the figure that as the crowd density increases, the interval with a larger proportion of the probability gradually moves in the direction of increasing acceleration. The distribution of the peak acceleration is concentrated in the middle region, with small proportions on both sides, obeying normal distribution. In addition, under the 0.4 P/m^2 working conditions, the probability that the peak acceleration is distributed in the 0.15 to 0.25 m/s^2 interval continues to increase. Similarly, the plural domain under the 0.6 P/m^2 working condition is distributed in the range of 0.2 ~ 3.5 m/s^2. Table 3 shows the cumulative probability distribution of the peak acceleration less than or equal to a certain value under various working conditions. In Table 3, when the crowd density increases to 0.4 and 0.6 P/m^2, the cumulative probability of peak acceleration less than 0.05 m/s^2 is significantly reduced to 8.42% and 0.95%, respectively. With the increase of crowd density and walking speed, the vibration comfort degree of the footbridge gradually decreases.

100% of pedestrians are predicted to perceive vibrations when the maximum acceleration reaches 0.35 m/s^2 Feng et al. (2013). Therefore, the peak acceleration of 0.35 m/s^2 is used as the limit of pedestrian comfort in this paper. Meanwhile, the inductive coefficient of pedestrians is set to 100%. Pedestrians are considered comfortable when the peak acceleration is below the threshold value. The probability of peak acceleration below the threshold value under different operating conditions is shown in Fig. 11.

In Fig. 11, the comfort probability values of crossing the footbridge at different walking speeds are 93.8%, 92% and 89.48% when the crowd density of the footbridge is 0.2 P/m^2. The comfort probability values of crossing the footbridge at different walking speed under 0.4 P/m^2 working conditions are 86.97%, 81.30%, and 72.55% respectively. When the crowd density increases to 0.6 P/m^2, the comfort probability at different walking speeds is significantly reduced to 63.83%, 58.57%, and 52.08%. The above analysis concludes that the effect of increasing crowd density at the same speed has a greater effect on the comfort probability values than different walking speeds at the same crowd density.

In real life, the crowd density and walking speed of pedestrian load are usually uncertain. Thus, the Monte Carlo random sampling method is used to generates 1000 groups of random pedestrian loads with normal distribution of walking speed, and the parameters of the footbridge remain unchanged. The generated 1000 pedestrian loads are used to calculate the vibration response of the footbridge, and then the peak acceleration statistics were obtained. The cumulative probability

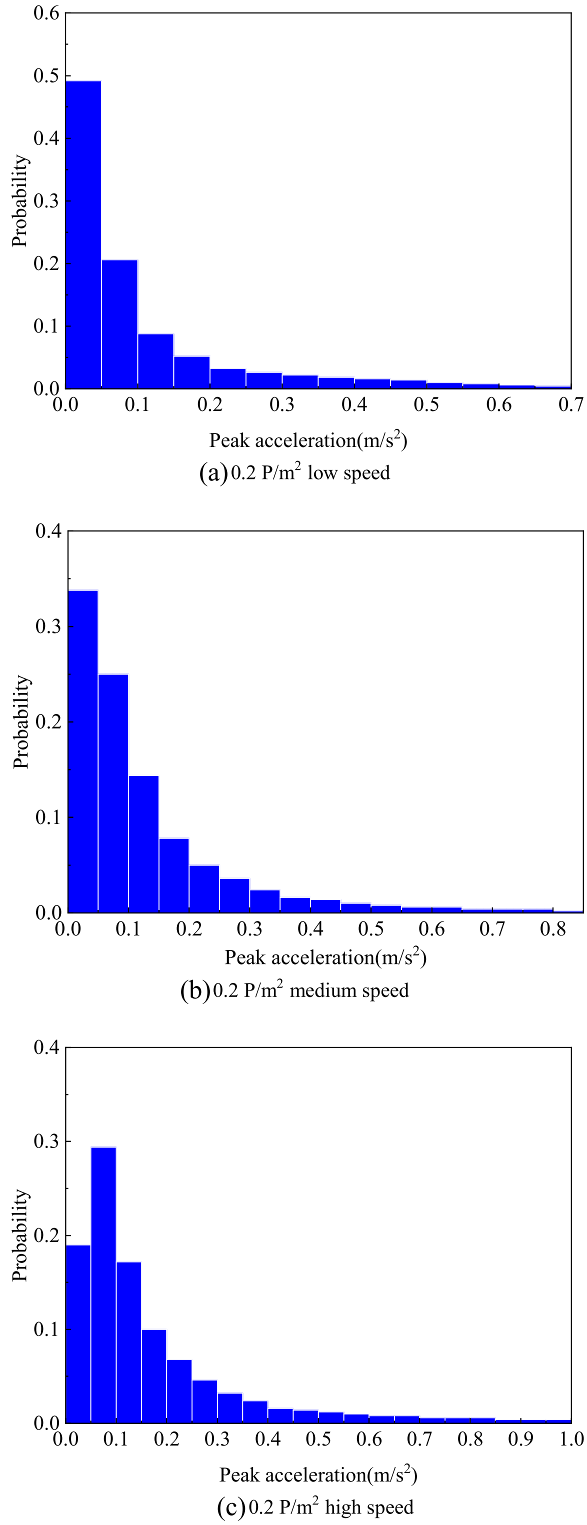
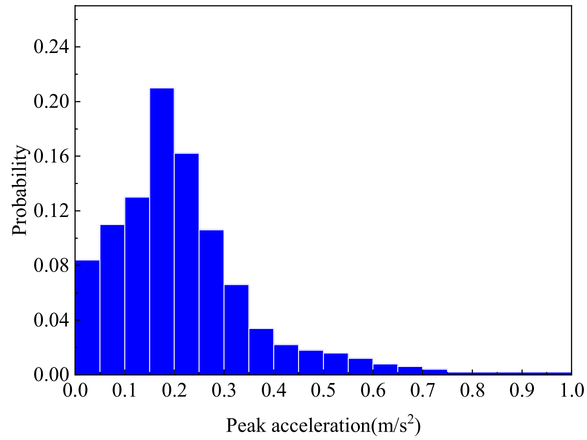
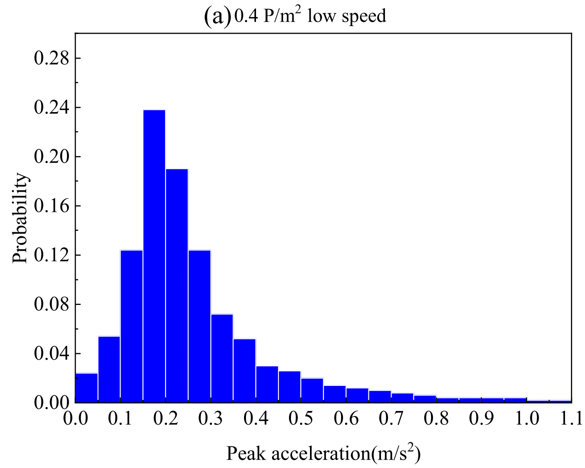


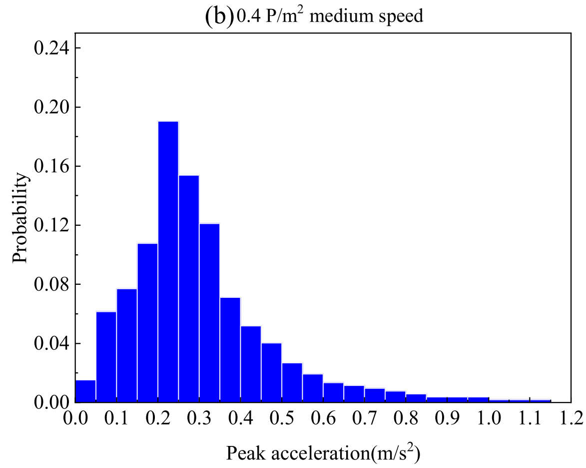
Fig. 8 Probability distribution of peak acceleration when the crowd density is 0.2 P/m^2



(a) 0.4 P/m^2 low speed



(b) 0.4 P/m^2 medium speed



(c) 0.4 P/m^2 high speed

Fig. 9 Probability distribution of peak acceleration when the crowd density is 0.4 P/m^2

distribution is shown in Fig. 12. Figure 12 indicates that the cumulative probability of the acceleration peak value is less than or equal to 0.35 m/s^2 under this working condition is 71.3%, which is extremely similar to the cumulative probability distribution of fast crossing the footbridge under 0.4 P/m^2.

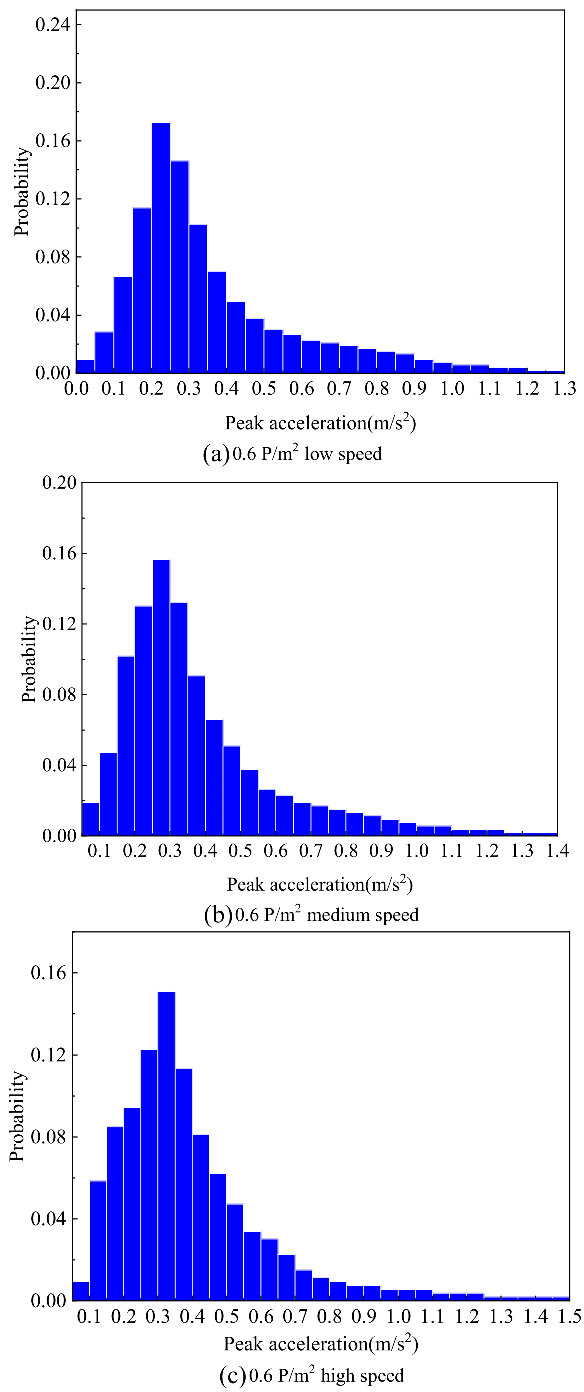


Fig. 10 Probability distribution of peak acceleration when the crowd density is 0.6 P/m^2

4 Comfort evaluation method and verification

This study proposes to use the cumulative probability that exceeds the acceleration limit (the proportion of pedestrians who feel uncomfortable.) as the comfort evaluation index. In order to verify the reasonableness of the proposed index, the improved annoyance rate comfort evaluation method of the literature (Chen 2019; Chen et al.

Table 3 Cumulative probability distribution table under various working conditions

Case	0.2 P/m^2			0.4 P/m^2			0.6 P/m^2		
a_{max}/m·s^{-2}	Low speed	Medium speed	High speed	Low speed	Medium speed	High speed	Low speed	Medium speed	High speed
0.05	49.20	33.80	18.85	8.42	2.36	1.54	0.95	0.00	0.00
0.10	69.80	58.80	48.02	19.44	7.68	6.38	3.79	1.88	0.94
0.15	78.60	73.20	65.08	32.46	19.88	15.36	10.42	6.59	6.79
0.20	83.80	81.00	75.00	53.51	43.31	26.10	21.78	16.76	15.28
0.25	87.00	86.00	81.75	69.74	62.01	45.11	39.02	29.76	24.72
0.30	89.60	88.60	86.31	80.36	74.21	60.46	53.60	45.39	36.98
0.35	93.80	92.00	89.48	86.97	81.30	72.55	63.83	58.57	52.08
0.40	93.60	93.01	91.87	90.38	86.42	79.65	70.83	67.61	63.40
0.45	95.20	95.00	93.45	92.59	89.37	84.84	75.76	74.20	71.51
0.50	96.60	96.00	94.84	94.39	91.93	88.87	79.55	79.28	77.74

2021) was referenced. The annoyance rate is the ratio of the number of people with annoyance responses to the number of people participating in the experiment at a certain vibration intensity, which is similar to the proposed index in this paper, and both use the form of probability. Therefore, the annoyance rate value can be used to compare with the proposed comfort index. By calculating the annoyance rate value corresponding to the peak acceleration of each group of working conditions, and averaging, the annoyance rate in this mode is obtained, as shown in formula (19):

$$\bar{A} = \sum_{i}^{N} \frac{A(x=i)}{N} \tag{19}$$

Where $A(x=i)$ is the annoyance rate at the i-th vibration response.

The discomfort probability values and the calculated annoyance rate values are shown in Table 4. Table 4 denotes that the errors between the proposed discomfort probability values and the calculated annoyance rate results are within 6%, which verifies the rationality of the method and provides a reference value for evaluating pedestrian comfort.

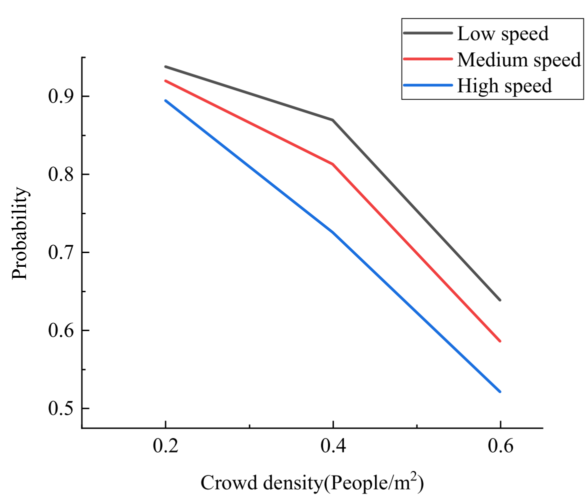


Fig. 11 Comfort probability value under various working conditions

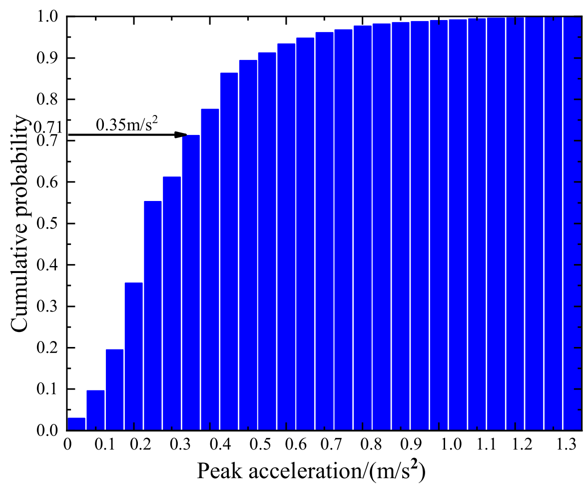


Fig. 12 Cumulative probability as result of MC

5 Conclusion

In this study, the analytical expression of the structural vibration response under the excitation of the multi-pedestrian load was derived. By introducing reasonable assumptions, the calculation method of the human-induced vibration response of footbridges was obtained. Then, the change law of structural dynamic characteristics under different crowd densities was studied. Based on the results of statistics, a comfort evaluation index was proposed. Above all, the major conclusions drawn from the research presented in this paper include the following:

The verification results in Fig. 7 and Table 2 indicate that the calculation result of formula (14) is in good agreement with the numerical method and the experimental results. The analytical method improves the calculation efficiency of structural dynamic response on the basis of satisfying the accuracy. Nevertheless, actual footbridge structures are generally complex, the analysis results based on the simply supported beam model are difficult to apply to all types of footbridges. The proposed analytical method provides a reference for the vibration response calculation of similar footbridges.

This study found that changes in crowd density have a greater impact on structural vibration response than walking speed. The cumulative probabilities of accelerations less

Table 4 Comparison of the discomfort probability values and annoyance rate values

Case		Discomfort probability	Annoyance rate calculation
0.2 P/m^2	Low speed	0.082	0.077
	Medium speed	0.08	0.086
	High speed	0.105	0.11
0.4 P/m^2	Low speed	0.13	0.142
	Medium speed	0.197	0.207
	High speed	0.275	0.284
0.6 P/m^2	Low speed	0.362	0.365
	Medium speed	0.414	0.408
	High speed	0.48	0.466
Random pedestrian		0.287	0.278

than or equal to 0.35 m/s^2 when passing the footbridge at medium speed under the three crowd densities were 92.0%, 81.3%, and 58.5%, respectively. When the crowd density is 0.2 P/m^2, the footbridge vibration comfort level is the highest.

The cumulative probability that exceeds the acceleration limit is suggested as a comfort evaluation index. By comparing with the results of previous studies, the proposed comfort evaluation method was proved to be reasonable. However, a large number of experiments are needed to demonstrate the practicality of this evaluation method, and the determination of discomfort probability thresholds requires further research.

Abbreviations

a_{max} Peak acceleration
a_{rms} Root mean square acceleration

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Authors' contributions
Yuhao Feng: Writing original draft, Formal analysis. Deyi Chen: Project administration, Review. Zhenyu Wang: Experiment. Shiping Huang: Data processing. Yuejie He: Editing. All authors read and approved the final manuscript.

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Availability of data and materials
Supplementary data to this article can be received from the corresponding author on reasonable request.

Declarations

Competing interests
The authors declare that there is no conflict of interest regarding the publication of this paper.

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